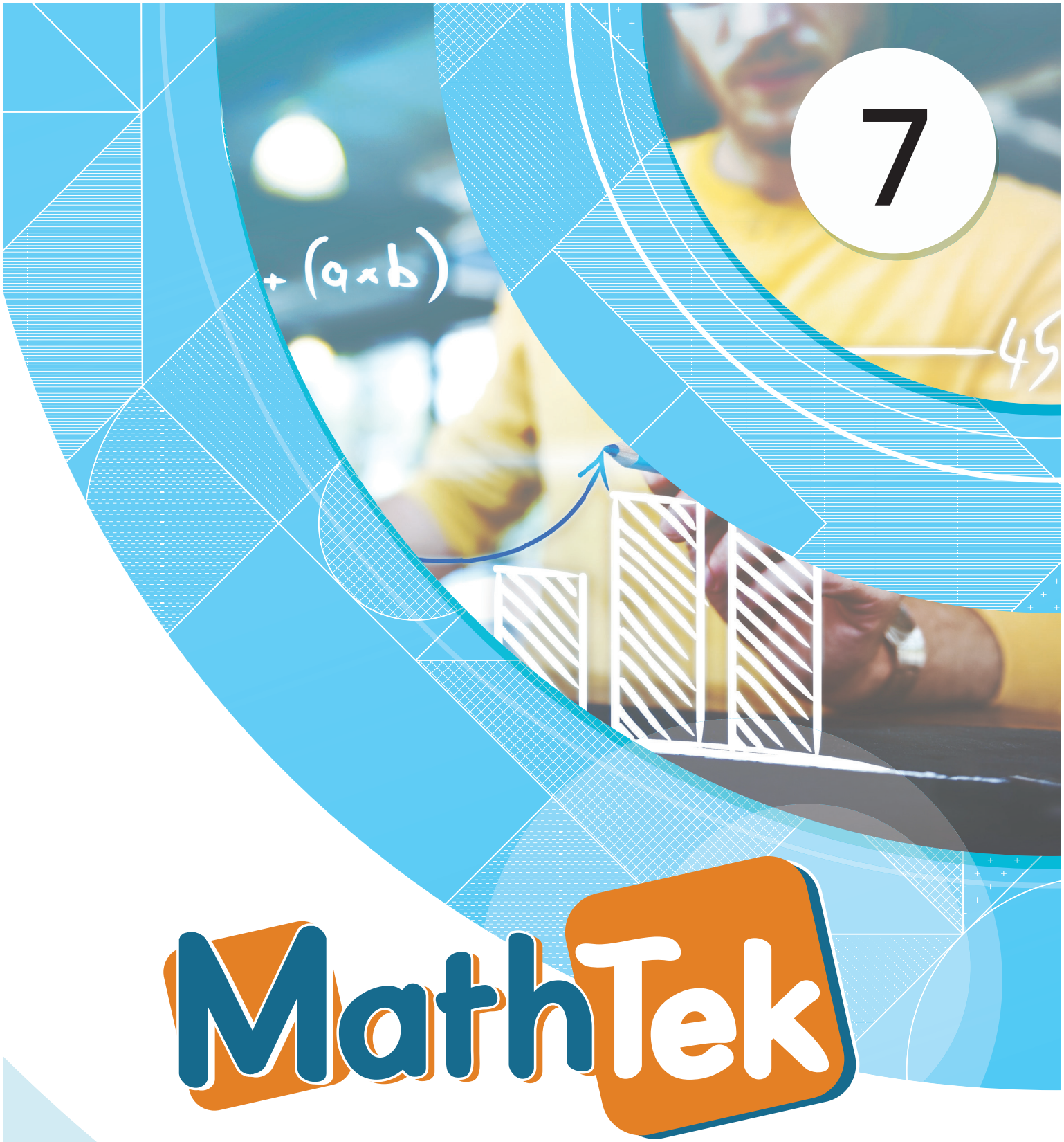

MathTek

Authors Janica M. Cabidoy
Gene Ethel P. Herman, LPT
Engr. Lennard Joseph U. Laya

Aligned with MATATAG Curriculum



Third Edition

Janica M. Cabidoy

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Engr. Lennard Joseph U. Laya

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FOREWORD

Learning Math in today's world is critical. Mathematical and analytical skills are not only necessary in specialized fields but also in everyday activities, it goes beyond the classroom and the school. It helps us analyze the information we are presented with, make sound judgments, and find solutions to the problems we encounter.

Learning Mathematics has been very challenging since students think that Math is difficult and that they would not really need it eventually. Some students even have developed anxiety towards mathematics because they have not mastered the basic concepts and skills yet. Because of these, TechFactors created the MathTek K-12 series based on the Department of Education MATATAG Curriculum. The Concrete-Pictorial-Abstract (CPA) Approach employed in MathTek K-6 is no longer necessary in instructing students of higher levels. Instead, a Problem-Based Learning (PBL) Approach is implemented in MathTek 7-10. The students will actively engage in problem solving to utilize, combine, and connect their previous knowledge and experiences to make sense of a new concept. The problems and examples presented lie within the real-life context to stimulate students' interest in mathematics. The use of contexts in the presentation of the lessons will enable the students to make connections between abstract concepts and real-life issues and situations.

TechFactors acknowledges the fact that not all students learn the same way, thus, alternative learning and instruction strategies are implemented. This technique of presenting lessons, and assessing the learning is diversified taking into consideration the different learning styles of the individual students.

With MathTek, we hope for students to develop solid foundations of math skills, enabling them to become critical thinkers and problem solvers ready to face the challenges of our ever-changing world.

TechFactors Inc.

How to Use This Book

The sections in the series and their brief descriptions are as follows:

 **Objectives**	states the target learning goals of the students.
 **Examine**	presents real-life situations related to the topic. This links mathematics and everyday life.
 **Recall**	pre-assesses student's knowledge of the concept of the topic using questions.
 **Explore**	introduces and discusses mathematical concepts through examples. It provides alternative methods of solving the examples.
 **Try This**	provides simple exercises that serve as guided practice with self-check/assessment questions.
 **Key Points**	provides a summary of the key ideas and important definitions discussed in the lesson.
 **Do**	includes exercises of varying levels of difficulty that assess the learners' conceptual understanding, computational and procedural fluency, critical-thinking skills, and problem-solving skills.
 **Brainstorm**	gives a pair activity to evaluate the skills of the students.
 **Design**	offers a group activity for students to work on collaboratively.
 **Fun Facts**	offers students interesting trivia related to the topic of mathematics in general.
 **Interactives**	shares the link where students can access alternative learning through virtual games.

What's New?

As we dedicate ourselves to improve the program, we have included the following changes.

INSTRUCTIONAL DESIGN

1. Aligned to the new **MATATAG curriculum** from the Department of Education
2. Concise lessons for clearer instruction
3. More and updated real-life situations on the practical applications of the topics
4. Additional and improved guided questions to solve problems
5. More and revised individual, pair, and group activities
6. Added interesting and relatable trivia
7. Integrated performance tasks with other subjects
8. New and improved interactives

LOOK AND FEEL

1. New layout and design
2. New vibrant illustrations

Embark on a dynamic educational journey with MathTek's latest offerings, where learning is made a great experience!

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TABLE OF CONTENTS

Unit 1 **1**

Lesson 1 Drawing Lines and Angles 3
Lesson 2 Drawing Polygons 13
Lesson 3 Angles and Polygons 21
Lesson 4 Problem Solving Angles and Polygons 29
Lesson 5 Application of Percentages 37
Lesson 6 Use of Rates 46
Lesson 7 Rational Numbers 55
Performance Task 65

Unit 2 **67**

Lesson 1 Square Roots and Cube Roots 69
Lesson 2 Irrational Numbers 76
Lesson 3 Units of Measure 85
Lesson 4 Volume of Cylinders 94
Lesson 5 Volume of Pyramids 102
Lesson 6 Sets 111
Lesson 7 Real Numbers 120
Performance Task 129

Unit 3 **131**

Lesson 1 Collecting Data 133
Lesson 2 Graphing Data 142
Lesson 3 Interpreting Data 150
Lesson 4 Integers 158
Lesson 5 Working with Integers 165
Lesson 6 Advanced Operations with Integers 174
Lesson 7 Absolute Value of Integers 182
Performance Task 191

Unit 4 **193**

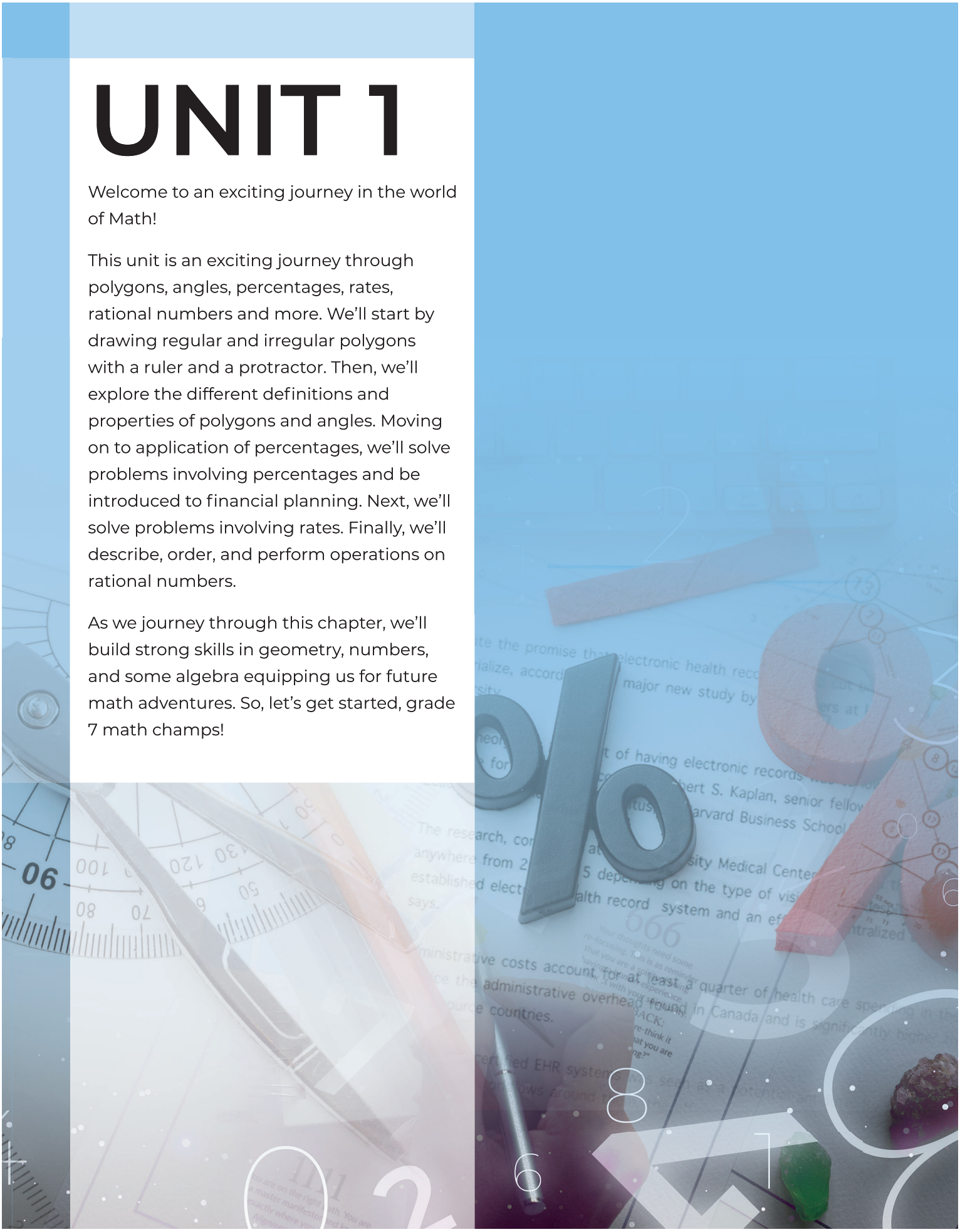
Lesson 1 Introduction to Algebra 195
Lesson 2 Algebraic Expressions 203
Lesson 3 Algebraic Equations 211
Lesson 4 Problem Solving Algebra 220
Lesson 5 Outcomes 229
Lesson 6 Problem Solving Outcomes 238
Lesson 7 Scientific Notation 247
Performance Task 255

UNIT 1

Welcome to an exciting journey in the world of Math!

This unit is an exciting journey through polygons, angles, percentages, rates, rational numbers and more. We'll start by drawing regular and irregular polygons with a ruler and a protractor. Then, we'll explore the different definitions and properties of polygons and angles. Moving on to application of percentages, we'll solve problems involving percentages and be introduced to financial planning. Next, we'll solve problems involving rates. Finally, we'll describe, order, and perform operations on rational numbers.

As we journey through this chapter, we'll build strong skills in geometry, numbers, and some algebra equipping us for future math adventures. So, let's get started, grade 7 math champs!



LESSON 1

Drawing Lines and Angles



Objectives

- Copy and bisect line segments and angles using a ruler and compass.
- Draw perpendicular and parallel lines.

Examine

When we look at buildings or structures, we often see corners where two walls meet. Have you ever thought about how those corners are formed? How can we create a perfect right angle (90-degree angle) at the intersection of two lines, and what tools might we use to do that? In this lesson, we'll be learning about constructing lines and angles.

Recall

Encircle the letter of the correct answer.

1. What is the first step in bisecting a line segment?
   a. Drawing a circle around the line
   b. Construct an arc using a compass
   c. Marking the midpoint of the line
   d. Drawing perpendicular lines on the ends

Polygons

- Drawing Lines and Angles
- Drawing Polygons
- Angles and Polygons
- Problem Solving Angles and Polygons

Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).

2. To bisect an angle using a compass, where should you first place the compass point?

   a. At the vertex of the angle
   b. Outside the angle
   c. On one of the arms of the angle
   d. In the middle of the angle's arms

3. How many points do two perpendicular lines intersect at?

   a. None
   b. One
   c. Two
   d. Infinite

4. When two lines are parallel, they:

   a. Intersect at one point
   b. Intersect at two points
   c. Never intersect
   d. Are always the same length

5. What does it mean to bisect a line segment?

   a. Draw it twice
   b. Measure its length
   c. Divide it into three equal parts
   d. Divide it into two equal parts

6. When bisecting an angle, the bisector will:

   a. Divide the angle into three equal parts
   b. Divide the angle into two equal parts
   c. Make the angle larger
   d. Make the angle smaller

7. You are given a line segment that is 8 cm long. Using your compass and ruler, how would you construct a line segment that is exactly half this length?

   a. Set the compass to 8 cm and draw an arc
   b. Set the compass to 4 cm and draw an arc
   c. Measure 4 cm on the ruler and draw a line
   d. Set the compass to 2 cm and draw an arc

8. John has a line segment XY that is 6 cm long. He wants to draw another line segment that is 1.5 times the length of line XY. Using his ruler, what should be the length of his new line segment?

   a. 3 cm
   b. 9 cm
   c. 12 cm
   d. 15 cm

9. Which of the following is not needed to copy a line segment accurately?

   a. Ruler
   b. Compass
   c. Pencil
   d. Protractor

10. A student uses a compass to copy an angle and notices the copy is larger than the original. Which could be the most likely reason for this discrepancy?

a. The student used a ruler instead of a compass
b. The compass was set to a wider or narrower distance than needed when copying the angle's sides
c. The student used a protractor to measure the angle
d. The original angle was an obtuse angle



Explore

Before we go to drawing polygons, we draw lines and angles first.

Mathematicians use a compass and a ruler in geometric construction. These are some basic geometric constructions.

The first construction is copying a segment.

Given: Segment DE



Objective: To construct a segment congruent to the given segment DE

Steps:

1. Draw a line using a ruler. Choose any point on the line and label it point F.



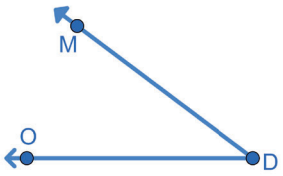
2. Using a compass, place the pin to point D and stretch the compass to the width of line segment until point E. Place now the compass pin at point F, then construct an arc that intersects the line at point G.



Result: line FG is congruent to line DE

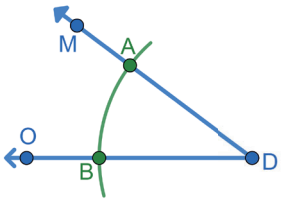
The second construction is copying an angle.

Given: angle MDO

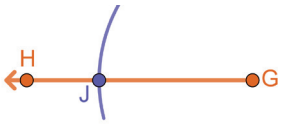


Objective: To construct an angle congruent to angle MDO

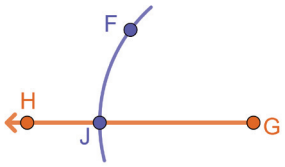
Steps:



1. Place a compass pin at point D. With a suitable compass opening, construct an arc, which intersects the sides of ∠MDO. Label these points of intersection point A and point B respectively.

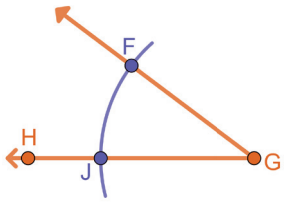


2. Using a ruler, draw ray GH. Place the compass pin at point G. With the same compass opening as in step 1, construct an arc. Label the point of intersection of the arc and ray GH as point J.



3. Open the compass to the length of line AB. Then, place the compass pin at point J, construct an arc which intersects the arc you drew earlier through point J, then label the point of intersection as point F.

4. Using a ruler, draw ray GF.



Result: ∠FGH ≅ ∠MDO

The third construction is a perpendicular bisector of a segment.

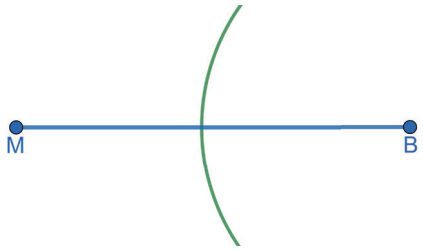
Given: Segment MB



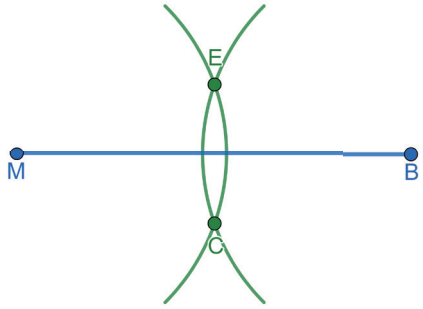
Objective: To construct a perpendicular bisector of segment MB

Steps:

1. Construct an arc by placing the compass pin at point B. Make sure that the compass opening is greater than half of line MB but less than line MB.

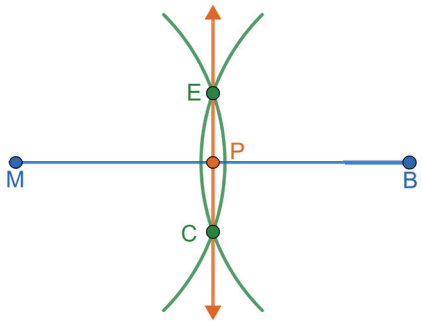


2. Construct another arc that intersects the previous arc by placing the compass pin at point M, with the same compass opening. Label the intersections as point E and point C.



3. Draw line EC.

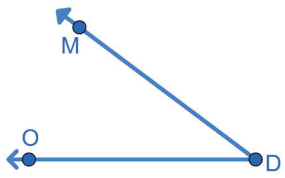
   Note: P is the midpoint of the segment MB



   Result: line EC is the ⊥ bisector of line MB

The fourth construction is a bisector of an angle.

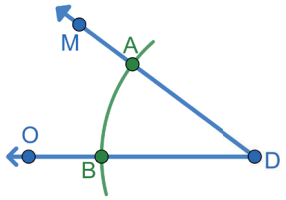
Given: angle MDO



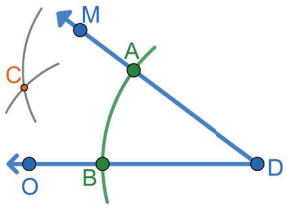
Objective: To bisect angle MDO

Steps:

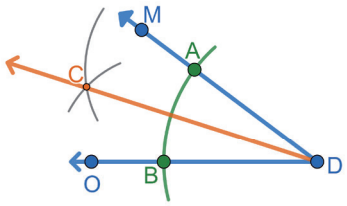
1. Draw an arc that intersects line DM and line DO, by placing the compass pin at point D and with a suitable compass opening. Label the intersections point A and point B respectively.



2. Construct an arc with a compass opening greater than half of line AB but less than line AB, by placing the compass pin at point A. Construct an arc that will intersect the previous one, using the same compass opening by placing the compass pin at point B. Label the point of intersection as point C.



3. Using a ruler, draw ray DC.



Result: ray DC is the angle bisector, angle MDC and angle CDO are congruent

The fifth construction is perpendicular lines.

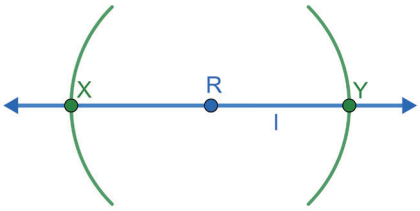
Given: Line l and point R at l



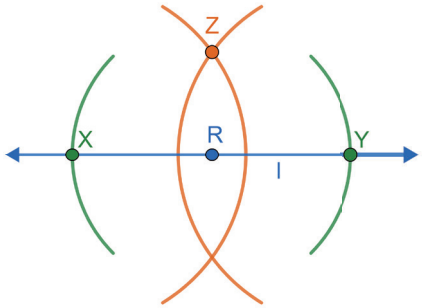
Objective: To construct a line perpendicular to l through R

Steps:

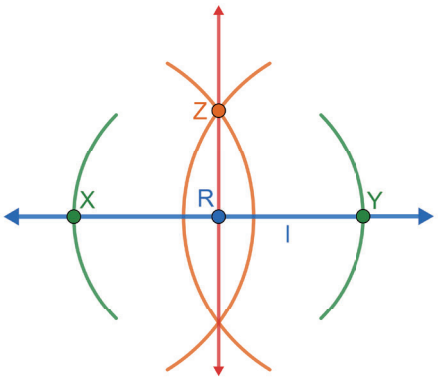
1. Construct two arcs that intersect line l, by placing the compass pin at point R, with a suitable compass opening. Label the points of intersection as point X and point Y.



2. Construct an arc with a compass opening of more than line RY but less than line XY by placing the compass pin at point X. Construct another arc that intersects the previous one by using the same compass opening and placing the compass pin at point Y. Label the intersection point Z.



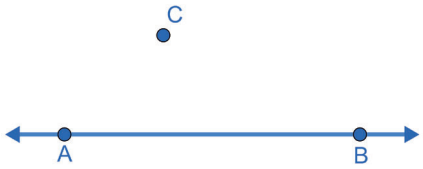
3. Draw line RZ.



Result: line RZ is perpendicular to line XY.

The last construction is parallel lines.

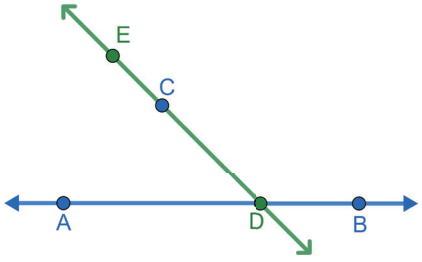
Given: Line AB and point C



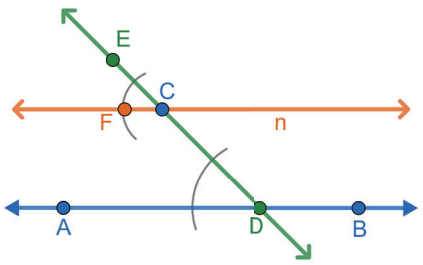
Objective: To construct a line through C parallel to line AB

Steps:

1. Draw a line through point C that intersects the given line AB at any point D.



2. Construct $\angle ECF$ by placing the compass pin at point C and by copying $\angle CDA$ (use the steps in copying an angle).



Result: line AB is parallel to line n



Try This

Now it is your turn to draw. You need paper, a ball pen, a ruler, and a compass. Draw a horizontal line of any length. Draw a line parallel to it.

1. After drawing your horizontal line, what can you tell us about its orientation? How do you know it's horizontal, and what does "horizontal" mean in terms of lines and angles?
2. When you draw a line parallel to the horizontal one you just made, what should the relationship between the two lines be? How can you use your ruler and compass to ensure that the second line is parallel to the first?

Note that geometry is not just about rigid rules. It can also be a creative endeavor. In fields like art and design, geometric principles can be applied to create visually stunning and structurally sound work.

You are now experts in drawing lines and angles. In the next lesson, you will be drawing polygons!



Key Points

- The use of a ruler and compass help in copying a line segment and angle measure.
- Bisecting a line segment or an angle measure means dividing it into 2 equal parts.
- Parallel and perpendicular lines can be drawn precisely using a ruler and compass.



Do

Encircle the letter of the correct answer.

1. Which property best describes two lines that never intersect?

   a. Perpendicular b. Segment c. Ray d. Parallel

2. Which of the following tools is NOT used for bisecting a line segment?

   a. Protractor b. Compass c. Ruler d. Pencil

3. What tool is essential for bisecting a line segment?

   a. Eraser b. Compass c. Protractor d. Highlighter

4. When bisecting an angle, you create...

   a. Two equal angles
   b. Two different angles
   c. A straight line
   d. A perpendicular line

5. What does "perpendicular" mean in geometry?

   a. Lines that never meet
   b. Lines that meet at a right angle
   c. Lines that meet at an acute angle
   d. Lines that are the same length

6. When copying an angle using a ruler and compass, the newly drawn angle should be...?

   a. Larger than the original
   b. Smaller than the original
   c. Equal to the original
   d. Inverted from the original

7. What is the result when you bisect a 60-degree angle?

   a. 30 degrees b. 45 degrees c. 60 degrees d. 120 degrees

8. In which profession might you use the skills of copying and bisecting lines and angles?

   a. Chef b. Singer c. Architect d. Nurse

9. Why do we bisect lines and angles in geometry?

   a. To make them shorter
   b. To divide them into two unequal parts
   c. To replicate them
   d. To divide them into two equal parts

10. Which of these is NOT a reason to study geometry?

    a. Enhancing spatial awareness
    b. Learning to draw
    c. Developing problem-solving skills
    d. Understanding the natural world



Brainstorm

Your teacher will assign you a partner. Now it is your turn to bisect a segment and an angle. Get your ball pen, paper, ruler, and compass. Draw any segment and angle on your paper. Exchange papers with your partner. Bisect the segment and angle of your partner.



Design

Your teacher will group you in teams of 4 members each. Form a circle. Get your ball pen, paper, ruler, and compass. Draw a line of any length using your ruler. Label the 2 endpoints with the initials of your first name and last name. Pick another letter if the 2 letters are the same. Pass the paper clockwise. Use your compass to draw a congruent line at the bottom of your classmate's work. Label the 2 endpoints with your initials. Repeat this process until all 3 of your groupmates have copied the length of your line segment. Check their answers.



Fun Facts

One of the most important real-life applications of this geometric construction is our house. Without the fundamental and basic geometric constructions, our house would not be built like what we have now or before. Simply by having a building plan for a house, either it's a plan made by an engineer or an architect or a simple sketch from a carpenter, still it has something to do with the geometric construction.



LESSON 2

Drawing Polygons



Objectives

- Describe polygons.
- Draw polygons using a ruler and protractor.
- Draw polygons with given angle measures.

Examine

When we see a city skyline made of buildings, we discover that they are made of various shapes that are essentially polygons. Architects had to carefully consider the angles and dimensions of each structure to create a visually appealing and functional cityscape. Have you ever wondered how these polygons were formed? In this lesson, we will discuss about constructing polygons.

Polygons

Drawing Lines and Angles

Drawing Polygons

Angles and Polygons

Problem Solving Angles and Polygons

Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).

Recall

Encircle the letter of the correct answer.

1. What is a polygon?
   a. A type of circle
   b. A 3D object
   c. An open curve
   d. A closed plane figure formed by three or more line segments

2. Which polygon has exactly three sides?

   a. Square b. Triangle c. Hexagon d. Pentagon

3. How many sides does an octagon have?

   a. 6 b. 8 c. 7 d. 9

4. What is a vertex in a polygon?

   a. The center of the polygon
   b. The point where two line segments meet
   c. The edge of the polygon
   d. The middle point of a line segment

5. What is a diagonal in a polygon?

   a. A line segment connecting two non-consecutive vertices
   b. The perimeter of the polygon
   c. A side of the polygon
   d. An exterior angle

6. A rhombus is a type of...

   a. Triangle b. Pentagon c. Quadrilateral d. Hexagon

7. Which of the following is NOT a type of quadrilateral?

   a. Parallelogram b. Rhombus c. Hexagon d. Rectangle

8. Which polygon will always have angles that sum up 540°?

   a. Triangle b. Quadrilateral c. Pentagon d. Hexagon

9. Which polygon has all sides and angles equal?

   a. Regular polygon b. Irregular polygon c. Diagonal polygon d. Concave polygon

10. Which of the following has the LEAST number of sides?

    a. Hexagon b. Heptagon c. Triangle d. Pentagon



Explore

When you were in grade 4, you discussed about how to draw angles using a protractor and ruler. We are going to use this skill and the tools to draw polygons.

The word polygon is derived from two Greek words. Poly means many and gon means angle. A polygon is a plane figure formed by three or more segments such that each segment unites at endpoints. Note that each endpoint is shared by only two noncollinear segments.

Remember that polygons are named according to the number of sides and angles. Here are some of the types of polygons.

Polygon	Number of Sides and Angles
Triangle	3
Quadrilateral	4
Pentagon	5
Hexagon	6
Octagon	8
Decagon	10

Here are the parts of a polygon. The side is 1 of the line segments that compose the polygon. The vertex is a point where 2 line segments meet. The diagonal is a line connecting 2 non-consecutive vertices.

Here are some situations that show the importance of polygons. Daisy likes board games and she would like to create an additional pentagonal tile for one game. Janica wants to create a quilt with a pentagon pattern. Ralph likes designing graphics using his computer and would like to understand the basics of shapes by manually drawing a pentagon.

Let us show these students how to draw an irregular pentagon first. An irregular pentagon is closed shape with 5 unequal sides. Take note that some sides can be equal in length but not all 5 sides are equal. The other polygons can also be irregular.

Here's how to draw an irregular pentagon:

Materials:

- A ruler (for straight lines)
- A protractor (if specific angles are needed, but not necessary for a random irregular polygon)
- A pencil
- Paper

Steps:

1. Start with a Point:

   Draw a point on your paper. This will be the starting point of your polygon.

2. Draw the First Line:

   From your starting point, draw a line in any direction. This will be the first side of your irregular polygon. You can make it as long or as short as you like.

3. Choose an Angle:

   If you're using a protractor and want specific angles, place the protractor's midpoint over the endpoint of the line you just drew. Choose an angle that isn't 180° (as that would just give a straight line) and mark it. If you're not using a protractor, simply choose a random direction in which to draw your next side.

4. Draw the Next Line:

   From the endpoint of your first line, draw another line starting at the angle you chose. Again, this line can be of any length, but not so long that it crosses your original line.

5. Continue Drawing Lines:

   Repeat steps 3 and 4, choosing various lengths and angles for each side, until you have as many sides as you want for your irregular polygon.

6. Close the Polygon:

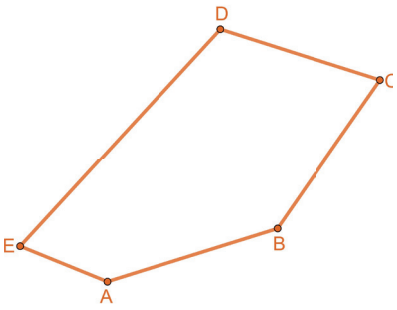
   Once you're satisfied with the number of sides, draw a final line that connects the endpoint of your last line to the starting point.

7. Review:

   Look over your polygon to ensure all sides are connected and that it fits the definition of a polygon (a closed shape with straight sides).

   Remember, the idea behind an irregular polygon is that it doesn't have a consistent pattern in terms of side lengths or angles, so feel free to be as random or as structured as you want in your drawing process.

Here is a sample of an irregular pentagon.



The same steps can be applied to any other irregular polygon. The only difference is the number of sides.

Daisy, Janica, and Ralph now would like to draw a pentagon with equal sides and interior angles. What they want is called a regular pentagon where the sides and angles are equal. The other polygons can also be regular.

Let us help students to a draw a regular pentagon. Using a ruler and a protractor requires precision to ensure that all sides and angles are equal. Here's a step-by-step guide to draw a regular pentagon:

Materials:

- A ruler
- A protractor
- A pencil
- Paper

Steps:

1. Draw the Base Line:

   Using your ruler, draw a straight horizontal line. This will be the base of your pentagon. Label the left endpoint as point A and the right endpoint as point B.



2. Determine the Interior Angle:

   A regular pentagon has five equal angles. The sum of interior angles for any polygon is given by (n - 2)(180°), where n is the number of sides. For a pentagon: (5 - 2)(180°) = (3)(180°) = 540°.

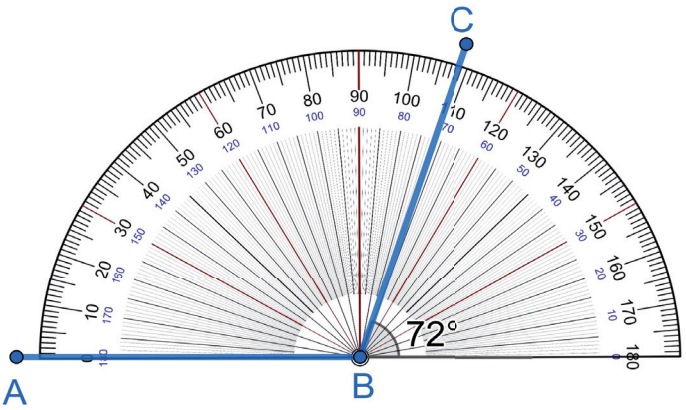
   Each interior angle of a regular pentagon is 540° ÷ 5 = 108°. However, to draw the next line using your protractor, you'll need to consider the external angle, which is 180° - 108° = 72°.

3. Construct the Second Side:

   Place the midpoint of the protractor at point B and align it so that the horizontal line lies over the 0° mark.

   Measure 72° from the baseline using the protractor and mark it.

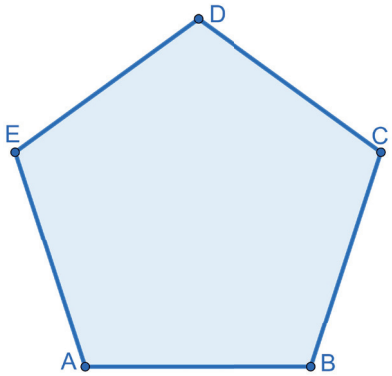
   Draw a straight line from point B to this mark, ensuring it's the same length as line AB. Label the endpoint of this line as point C.



4. Repeat for the Remaining Sides:

   Using point C as the new reference, repeat the process, always marking out the 72 external angle and drawing the next side to be equal in length to the previous sides.

   Continue this process until you have five sides. The fifth side should connect back to your original point A.



5. Adjustments:

   If the fifth side doesn't perfectly meet point A, it indicates a minor error in measurements or drawing. Such errors can accumulate, especially with manual tools. Double-check your angles and side lengths, and make small adjustments as needed.

   Note: Drawing a regular pentagon with precision requires practice, especially when using manual tools. It's common to encounter small errors, but with patience and attention to detail, the result will be a near-perfect regular pentagon.

   The same steps can be applied to any other regular polygon. The differences are the number of sides and the measures of the interior angles.



Try This

Now it is your turn to draw. You need a ruler and a protractor. Draw a regular hexagon. This polygon has 6 equal sides and interior angles. Note that the sum of interior angles for any polygon is given by $(n - 2)(180°)$, where n is the number of sides. Use $n = 6$. After getting the sum, divide by 6 to get the measure of one interior angle. Show the step-by-step illustrations with explanations.

1. What is the difference between an irregular hexagon and a regular one?
2. What is the sum of the interior angles of a hexagon?
3. What is the measure of one interior angle of a hexagon?

Those regular hexagons drawings look great! You can follow the same steps to create other polygons. Try to draw a regular octagon and a regular decagon too.

You may go to the internet and go to geogebra.org. The GeoGebra software helps in creating drawings of polygons and other figures. You can do the illustration more accurately and in less time. Try it out!



Key Points

- A polygon is a closed, two-dimensional shape with straight sides. Polygons can be categorized based on their sides and angles.
- There are two types of polygons. For regular polygons, all the sides and angles are equal. For irregular polygons, the sides and angles aren't uniformly equal.
- A ruler is a tool used to draw straight sides and a protractor is used to draw specific angle measures.
- There are steps to draw regular and irregular polygons by hand.



Do

Encircle the letter of the correct answer.

1. How many sides does a hexagon have?

   a. 5 b. 6 c. 7 d. 8

2. What is a polygon with four sides called?

   a. Triangle b. Hexagon c. Pentagon d. Quadrilateral

3. If a polygon has 5 sides, what is its name?

   a. Heptagon b. Octagon c. Pentagon d. Decagon

4. Which of the following is NOT a characteristic of regular polygons?

   a. All angles are equal
   b. All sides are equal
   c. They have curved sides
   d. They are closed shapes

5. Which tool is best used to measure angles when drawing polygons?

   a. Ruler b. Compass c. Protractor d. Eraser

6. Which of the following shapes is NOT a polygon?

   a. Circle
   b. Rectangle
   c. Pentagon
   d. Trapezoid

7. What is the sum of the interior angles of a triangle?

   a. 90° b. 180° c. 270° d. 360°

8. If one angle of a regular pentagon is 108°, how much would all its angles sum up to?

   a. 360° b. 450° c. 540° d. 720°

9. Why are curved shapes not considered polygons?
   a. They have too many sides
   b. They have angles greater than 180°
   c. They are not closed figures
   d. Polygons have straight sides

10. Which of the following polygons can be irregular?
   a. Circle
   b. Triangle
   c. Ellipse
   d. Cube

II. Analyze and construct the following.

1. An equilateral triangle with a perimeter equal to 24 cm.
2. A regular hexagon with sides equal to 3 cm.
3. A square with sides equal to 10 cm.



Brainstorm

Your teacher will assign you a partner. Now it is your turn again to draw. You may want to use a ruler and a protractor. Both of you are to draw 3 irregular hexagons. This is a polygon with 6 unequal sides. Be creative. Compare your drawings after. A challenge is to draw a diagonal on your partner's hexagons.



Design

Your teacher will group you into teams with 4 members each. Take the learning outside! Each group is given a list of different polygons to find in the school or local environment. The team must work together to identify and sketch the polygons they encounter, whether it's a rectangular window, a triangular sign, or a hexagonal nut.



Fun Facts

Origami, the art of paper folding, often involves creating intricate polygons. Folding a square piece of paper can result in polygons like triangles, squares, pentagons, and more.

LESSON 3

Angles and Polygons



Objectives

- Describe angle pair relationships.
- Classify polygons.
- Deduce the relationship of exterior and interior angles of a polygon.
- Determine the measures of angles.

Examine

There are many examples of geometric figures that we can see everywhere, no matter where you look, and almost everything is made up of simpler geometry. One example is a bridge that we use to go to another place quickly. If you are to be an engineer and are tasked to design a bridge that will connect two of the biggest islands in your country, how will you design the bridge? Sketch your design, take note: A bridge must be strong and flexible enough to hold tons of vehicles that are passing through.

In this lesson, we are to describe and explain the relationships between angle pairs. This also includes the exterior and interior angles of a polygon. We are to classify polygons according to the number of sides, whether they are regular or irregular, and whether they are convex or non-convex. Lastly, our goal is to help you determine the measures of angles.

Polygons

- Drawing Lines and Angles
- Drawing Polygons
- Angles and Polygons
- Problem Solving Angles and Polygons

Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).



Recall

Encircle the letter of the correct answer.

1. Which term describes two angles that sum up to 90°?

   a. Supplementary
   b. Adjacent
   c. Complementary
   d. Vertical

2. Which polygon has exactly five sides?

   a. Hexagon
   b. Triangle
   c. Pentagon
   d. Quadrilateral

3. Which term refers to a polygon that has all sides and angles equal?

   a. Regular b. Irregular c. Concave d. Convex

4. Two angles that share a common vertex and side but don't overlap are called...?

   a. Complementary
   b. Vertical
   c. Adjacent
   d. Supplementary

5. Which is NOT a property of a regular polygon?

   a. All angles are equal
   b. All sides are equal
   c. Angles are supplementary
   d. Sides are congruent

6. Which polygon can sometimes have an angle greater than 180°?

   a. Convex b. Regular c. Concave d. Equilateral

7. What is the sum of the interior angles of a triangle?

   a. 90° b. 180° c. 270° d. 360°

8. Which polygon has its interior angles sum up to 720°?

   a. Triangle b. Quadrilateral c. Pentagon d. Hexagon

9. If one exterior angle of a regular polygon is 60°, how many sides does the polygon have?

   a. 3 b. 4 c. 5 d. 6

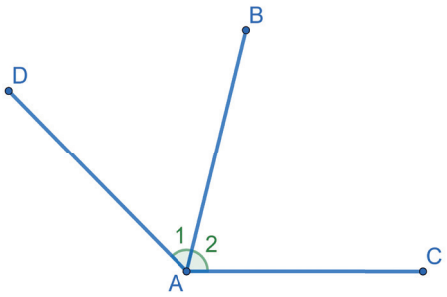
10. If the exterior angle of a regular polygon is 40°, how many sides does the polygon have?

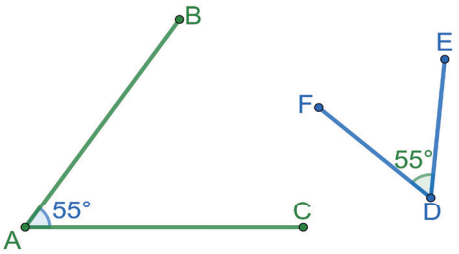
    a. 5 b. 7 c. 9 d. 10

Explore

When you were in grade 4, you learned about the different kinds of angles- right, acute, and obtuse. For this lesson, we are going to learn about angle pairs.

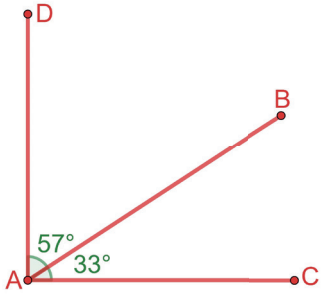
Adjacent angles are those that sit next to each other and share a common side and vertex. While these angles can have the same degree measure, they don't necessarily have to. Here is an illustration of adjacent angles. Angle 1 and angle 2 are adjacent.

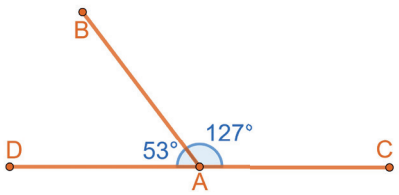




Congruent angles are angles with the same measure. Here is an illustration of congruent angles. Both angle BAC and angle EDF have measures of 55 degrees.

Complementary angles have 90° as their sum. Here is an illustration of complementary angles. The measure of angle DAB is 57° and the measure of angle BAC is 33°. $m\angle DAB + m\angle BAC = 57° + 33° = 90°$.

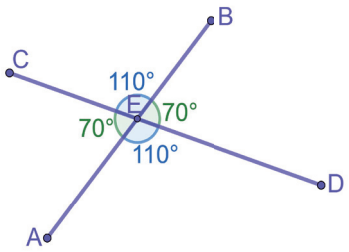




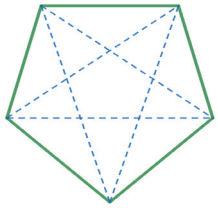
Supplementary angles have 180° as their sum. Here is an illustration of supplementary angles. The measure of angle DAB is 53° and the measure of angle BAC is 127°. $m\angle DAB + m\angle BAC = 53° + 127° = 180°$.

Linear pairs are a specific type of adjacent angles that are supplementary. The illustration shows a linear pair.

Vertical angles are formed by the intersection of two lines. They are opposite to each other and are always congruent. Angles CEA and AEC are vertical angles with a measure of 70°. Angles CEB and DEB are also vertical angles with a measure of 110°.



Other angle pairs like alternate interior angles and corresponding angles will be discussed in grade 9.



We discussed about polygons in the previous lesson. Here are some more of the types of polygons.

Polygon	Number of Sides and Angles
Triangle	3
Quadrilateral	4
Pentagon	5
Hexagon	6
Heptagon	7
Octagon	8
Nonagon	9
Decagon	10

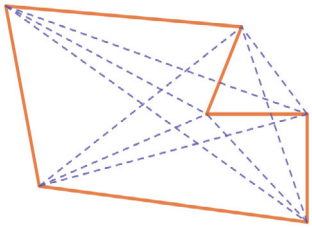
Recall from the previous lesson that a regular polygon has all its sides and angles congruent. While an irregular polygon does not have all sides and angles congruent.

The new concept in this lesson about polygons is if they are convex or non-convex. We also call non-convex polygons as concave polygons.

A convex polygon is a closed geometric shape in which all interior angles are less than 180 degrees. In other words, if you draw a straight line between any two points inside the polygon, that line will entirely lie inside the polygon. This is an illustration of a convex polygon. The diagonals are drawn and all of them lie inside.

In a convex polygon, all internal angles are "pointing" inward, meaning they face toward the center of the polygon. None of the angles "bulge out" or extend beyond the boundary of the shape.

A concave (non-convex) polygon is also a closed geometric shape, but it has at least one interior angle greater than 180 degrees. In other words, if you draw a straight line between certain points inside the polygon, that line may partially or entirely extend outside the polygon. Here is an illustration of a concave polygon.



In a concave polygon, there is at least one internal angle that "bulges out" or extends beyond the boundary of the shape. These angles are called reflex angles and have measures greater than 180 degrees.

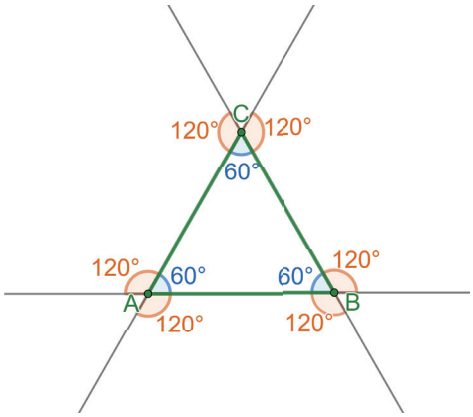
Remember from this previous lesson that the formula for getting one interior angle measure of a regular polygon is (n - 2)(180°) ÷ n where n is the number of sides. We now can see that this polygon is also convex.

If we have a regular and convex octagon, the measure of its interior angles is (8 - 2)(180°) ÷ 8 = 135 degrees.

We have been focusing on the interior angles of a polygon. If the polygon is regular, all the interior angles are equal. There is a relationship between the interior angle with the exterior one. The interior angle and its exterior angle form a linear pair. This means that these angles are adjacent and supplementary.

The formula to get the measure of an exterior angle of a polygon is 360° ÷ n where n is the number of sides. Note that the sum of all exterior angles is 360 degrees.

Let us use this formula to get the measure of the exterior angles of a triangle. 360° ÷ 3 = 120°. Let us show this in an illustration. The sides of the triangle are extended so that the exterior angles can be shown. The interior angles are 60 degrees and their corresponding exterior angles are 120 degrees. Adding 60 and 120 results in 180 degrees. Yes, the interior and exterior angles are supplementary.





Try This

For this activity, please get your drawing of a regular hexagon from the previous lesson. Extend the sides and use your protractor to measure the exterior angles. Just pick 1 of the 2 possible exterior angles of each interior angle. Check your measurements with the result if you use the formula of 180° - measure of the interior angle = measure of the exterior angle.

1. What is the measure of the exterior angle using a protractor?
2. What is the measure of the exterior angle using the formula?
3. What could be possible reasons if there is a difference in the angle measures using the protractor and the formula?



Key Points

- Angles are related to each other in several ways. We discussed about adjacent, congruent, complementary, supplementary, and vertical angles, as well as linear pairs.
- Polygons are classified according to the number of sides. A polygon can be regular or irregular. It can also be convex or concave. A convex polygon has all interior angles less than 180 degrees. A concave one has at least 1 interior angle greater than 180 degrees.
- Regular and convex polygons have interior angles that are computed using the formula, $(n - 2)(180°) \div n$ where n is the number of sides. The exterior angle is formed with 1 side and the extension of an adjacent side. The formula for the measure of exterior angles is $360° \div n$ where n is the number of sides of the polygon. The interior and exterior angles are supplementary.



Do

I. Encircle the letter of the correct answer.

1. Vertical angles are always:

   a. Adjacent
   b. Acute
   c. Complementary
   d. Congruent

2. A polygon with seven sides is called:

   a. Hexagon
   b. Heptagon
   c. Octagon
   d. Decagon

3. What is the sum of the exterior angles of any polygon?

   a. 90 degrees
   b. 180 degrees
   c. 270 degrees
   d. 360 degrees

4. Which of the following is an example of adjacent angles?

   a. Angles that sum up to 90 degrees
   b. Angles that are opposite when two lines cross
   c. Angles that share a common vertex and a side
   d. Angles that sum up to 180 degrees

5. What is the relationship between the interior and exterior angles at a single vertex of a polygon?

   a. They are complementary
   b. They are supplementary
   c. They are vertical angles
   d. They are congruent

6. Which of the following is a polygon with all sides of equal length and all angles of equal measure?

   a. Scalene triangle
   b. Irregular quadrilateral
   c. Regular pentagon
   d. Right triangle

7. The interior angle of a regular pentagon is:

   a. 108 degrees b. 120 degrees c. 100 degrees d. 72 degrees

8. A polygon has 6 sides. What is the sum of its interior angles?

   a. 720 degrees b. 900 degrees c. 540 degrees d. 1080 degrees

9. An exterior angle of a regular octagon is:

   a. Greater than its interior angle
   b. Less than its interior angle
   c. Equal to its interior angle
   d. Not related to its interior angle

10. How many sides does a polygon have if the sum of its interior angles is 1800 degrees?

    a. 12 sides b. 10 sides c. 14 sides d. 8 sides

II. Find the measure of one interior angle and the measure of one exterior angle for each polygon.

1. Octagon
2. Decagon
3. 15-sided polygon
4. 18-sided polygon
5. 21-sided polygon

III. List down 5 real-life examples of polygons. Make sure to show an illustration for each.

1.
2.
3.
4.
5.

IV. Answer the following questions.

1. How many sides does a polygon have if the sum of the interior angles is 3960 degrees?
2. How many sides does a polygon have if one exterior angle measures 36 degrees?
3. If you are going to design your house, which polygon would you use as the main structure of your house? Why?



Brainstorm

Your teacher will assign you a partner. It is time again to draw. You may want to use a ruler and a protractor. Both of you are to draw 2 convex pentagons and 2 concave pentagons. Be creative. Compare your drawings after. Draw the diagonals on your partner's pentagons to check if they are convex or concave.



Design

Your teacher will group you into teams of 4 members each. Play polygon Pictionary. Take turns drawing a polygon-related word without using any numbers or letters. The rest of the team has to guess the polygon being depicted.



Fun Facts

Video game graphics heavily rely on polygons to create three-dimensional environments. Game designers use polygons to construct characters, landscapes, and objects, allowing players to explore visually immersive virtual worlds.

LESSON 4

Problem Solving Angles and Polygons



Objectives

- Solve problems involving angles and polygons.

Examine

There are many real-life applications of angles and polygons. In which field are you interested in?

When planning the layout for floor tiles, designers often work with polygons. Rectangles, squares, and even hexagons are common tile shapes. Ensuring a seamless and aesthetically pleasing pattern involves manipulating these geometric forms.

Farmers often divide their fields into polygons to optimize irrigation systems and crop planting. The irregular shapes of fields, determined by natural boundaries and topography, impact decisions on water distribution and planting patterns.

Polygons

- Drawing Lines and Angles
- Drawing Polygons
- Angles and Polygons
- Problem Solving Angles and Polygons

Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).

Artisans crafting stained glass windows use polygons extensively. Each piece of colored glass is cut into various geometric shapes, and when assembled, they create intricate patterns and images.

Consider a basketball court or a soccer field. The key lines, three-point arcs, and goal areas all involve drawing polygons. Precision in designing these features ensures fair and accurate gameplay.

In this lesson, we are to use the concepts we have learned on angles and polygons to strategize on how to solve problems involving them.



Recall

Encircle the letter of the correct answer.

1. Which of the following is a polygon with 5 sides?
   a. Triangle b. Rectangle c. Pentagon d. Hexagon

2. How many sides does a hexagon have?
   a. Four b. Five c. Six d. Seven

3. What is the term for a closed shape with straight sides?
   a. Circle b. Curve c. Polygon d. Ray

4. What is the sum of the interior angles of a triangle?
   a. 90° b. 180° c. 270° d. 360°

5. When a straight line is divided into two angles and the two angles sum to 180°, what is the relationship between these angles?
   a. They are complementary
   b. They are vertical
   c. They are congruent
   d. They are supplementary

6. Which statement is true about the angles in a regular pentagon?
   a. All angles are less than 90°
   b. Each angle is exactly 90°
   c. Each angle measures more than 90° but less than 120°
   d. All angles are greater than 120°

7. If one angle in a triangle is 90°, and another is 45°, what is the third angle?
   a. 35° b. 45° c. 60° d. 75°

8. In a regular pentagon, what is the measure of each interior angle?
   a. 108° b. 114° c. 120° d. 126°

9. Which shape will NOT always have angles that add up to 360°?
   a. Rectangle b. Square c. Parallelogram d. Triangle

10. Two angles in a quadrilateral are 90° each. If the third angle is 60°, what is the fourth angle?
    a. 90° b. 100° c. 120° d. 130°

Explore

You have already learned much about angles and polygons. Let us continue to solve problems involving them.

Here are some problem-solving strategies we can use.

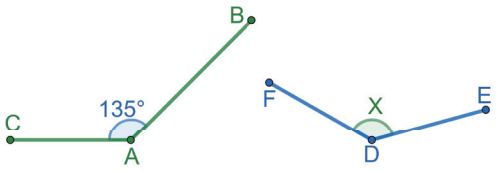
Drawing Diagrams: For many problems involving angles and polygons, drawing a labeled diagram can make it easier to understand and solve the problem. You were taught how to construct the illustrations.

Using Algebra: When working with angle relationships, it's often useful to set up equations to solve for unknown angles.

Use Properties and Theorems: Knowledge of properties of polygons (like all interior angles of a regular polygon being equal) can provide shortcuts in solving problems.

Here are some problems involving angle pairs.

Angle CAB and angle FDE are congruent angles. Angle CAB measures 135 degrees. What is the measure of angle FDE?



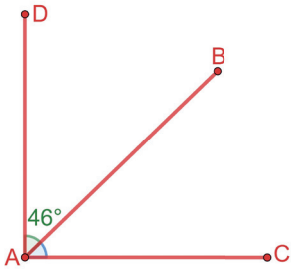
We write the measure of angle CAB equals 135 degrees in symbols as $m\angle CAB = 135°$.

Since the 2 angles are congruent, we write it as ∠CAB ≅ ∠FDE. Since congruent angles have the same angle measure, then m∠FDE = 135°.

The next problem is about complementary angles. Recall that the measures of these angles add up to 90 degrees.

Angle DAB and angle BAC are complementary angles. Angle DAB measures 46 degrees. What is the measure of angle BAC?

$$m\angle DAB + m\angle BAC = 90$$
$$46 + m\angle BAC = 90$$



We need to minus 46 from both sides of the equation to solve this.

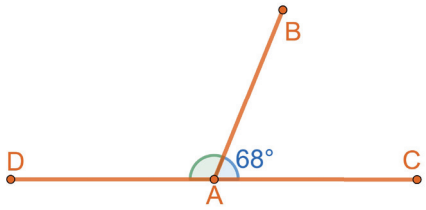
$$46 + m\angle BAC - 46 = 90 - 46$$
$$m\angle BAC = 44$$

The measure of angle BAC is 44 degrees.

The next problem is about supplementary angles. Recall that the measures of these angles add up to 180 degrees.

Angle DAB and angle BAC are supplementary angles. Angle BAC measures 68 degrees. What is the measure of angle BAC?

$$m\angle DAB + m\angle BAC = 180$$
$$m\angle DAB + 68 = 180$$



We need to minus 68 from both sides of the equation to solve this.

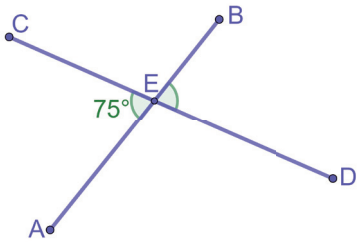
$$m\angle DAB + 68 - 68 = 180 - 68$$
$$m\angle DAB = 112$$

The measure of angle DAB is 112 degrees.

The last problem about angle pairs is about vertical angles. Remember that they are congruent.

Angle CEA and angle BED are vertical angles. Angle CEA measures 75 degrees. What is the measure of angle BED?

$$\angle CEA \cong \angle BED$$
$$m\angle CEA = 75°, \angle BED = 75°$$



The measure of angle BED is 75 degrees.

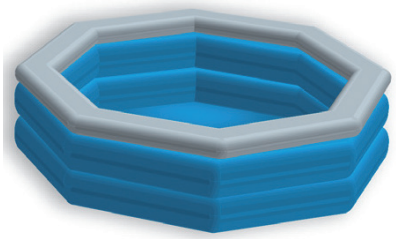
After solving problems on angle pairs, let us now tackle problems on angles of polygons.

Mr. Alvaran is planning to build a pool whose shape is a regular octagon. What is the measure of its interior angle?

Recall that the formula to find the interior angle of a regular polygon is given by $(n - 2)(180°) \div n$ where n is the number of sides.

An octagon has 8 sides.

$$(8 - 2)(180) \div 8$$
$$(6)(180) \div 8$$
$$1080 \div 8$$
$$135$$



The measure of the interior angle of a regular octagon is 135 degrees.

Architect Mortel is designing a hexagonal building. What is the sum of the measures of its interior angles?

Remember that the formula to find sum of the interior angles of a regular polygon is given by $(n - 2)(180^\circ)$ where n is the number of sides.

A hexagon has 6 sides.

$(6 - 2)(180)$ $(4)(180)$ 720	The measure of the sum of the interior angles of a regular hexagon is 720 degrees.

Here are some examples of finding the missing angle in irregular polygons.

Two interior angle measures of a triangle are 60 and 70 degrees. What is the measure of the third angle?

The sum of the interior angles of a polygon is given by $(n - 2)(180^\circ)$. A triangle has 3 sides so $n = 3$.

$(3 - 2)(180)$ $(1)(180)$ 180	The sum of the interior angles of a triangle is 180 degrees.

Let X be the angle measure we are looking for.

$$60 + 70 + X = 180$$

We subtract 60 and 70 from both sides of the equation to get the answer.

$60 + 70 + X - 60 - 70 = 180 - 60 - 70$ $X = 50$	The measure of the third angle is 50 degrees.

Luis has a study table. Three measures of the angles of a quadrilateral table are 120 degrees, 36 degrees, and 89 degrees. What is the measure of the fourth angle?

The sum of the interior angles of a polygon is given by $(n - 2)(180^\circ)$. A quadrilateral has 4 sides so $n = 4$.

$(4 - 2)(180)$ $(2)(180)$ 360	The sum of the interior angles of a quadrilateral is 360 degrees.

Let Y be the angle measure we are looking for.

$$120 + 36 + 89 + Y = 360$$

We subtract 120, 36, and 89 from both sides of the equation to get the answer.

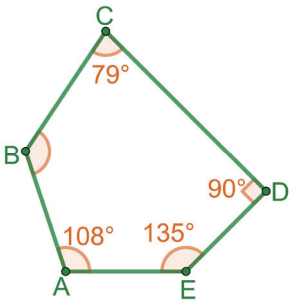
$$120 + 36 + 89 + Y - 120 - 36 - 89 = 360 - 120 - 36 - 89$$
$$Y = 115$$

The measure of the fourth angle is 115 degrees.



Try This

Given the figure, find the measure of angle ABC. Show your complete and step-by-step solution.



1. What is the sum of the measures of all the interior angles in a pentagon?
2. What is the measure of angle ABC?



Key Points

- A polygon is a closed figure with straight sides.
- For any polygon with n sides, the sum of its interior angles is given by the formula: $(n - 2)(180°)$.
- To find the measure of each interior angle in a regular polygon: $(n - 2)(180°) \div n$ where n is the number of sides.
- To solve problems, set up equations using the known properties of angles and polygons.



Do

I. Encircle the letter of the correct answer.

1. How many sides does a decagon have?
   a. 8 b. 10 c. 12 d. 14
2. How many angles does a pentagon have?
   a. 3 b. 4 c. 5 d. 6

3. A regular polygon has:
   a. All angles different
   b. All sides different
   c. Equal sides and equal angles
   d. No equal sides or angles

4. If two angles are complementary, their sum is:
   a. 90° b. 180° c. 360° d. 270°

5. What is the sum of the angles in a quadrilateral?
   a. 90° b. 180° c. 270° d. 360°

6. If one angle of a triangle is 90° and the second angle is 45°, what is the measure of the third angle?
   a. 45° b. 60° c. 30° d. 90°

7. In a rectangle, what is the measure of each interior angle?
   a. 90° b. 60° c. 45° d. 180°

8. In an equilateral triangle, what is the measure of each interior angle?
   a. 45° b. 60° c. 90° d. 120°

9. Which polygon has an interior angle sum of 720°?
   a. Triangle b. Quadrilateral c. Hexagon d. Pentagon

10. Which of the following statements is NOT true for all regular polygons?
    a. All sides are equal
    b. All angles are equal
    c. The interior angle sum is 180°
    d. The exterior angles sum up to 360°

II. Solve the following problems.

1. A regular hexagon has a side length of 6 cm. Calculate its perimeter.
2. If the interior angles of a pentagon measure 540 degrees in total, what is the measure of each angle?
3. If an octagon has interior angle measurements of 135 degrees each, what is the total sum of its interior angles?
4. What is the sum of exterior angles for a regular decagon?
5. In a parallelogram, one angle measures 50 degrees. Find the measurement of its adjacent angle.
6. The measures of two angles in a triangle are 35 degrees and 75 degrees. Find the measure of the third angle.
7. Calculate the exterior angle of a triangle if the two interior angles are 45 degrees and 60 degrees.

8. In a quadrilateral, three angles measure 70 degrees, 120 degrees, and 90 degrees. Find the measure of the fourth angle.
9. In a trapezoid, the two adjacent angles are 75 degrees and 105 degrees. Find the measure of the other pair of adjacent angles.



Brainstorm

Your teacher will assign you a partner. Assign someone to be person A and the other to be person B. Person A solves for the measure of 1 interior angle of a regular heptagon. Person B solves for the measure of the sum of all interior angles of a regular heptagon. Once both of you are done, discuss with one another how you get your answers. Find the relationship between the measure of 1 interior angle and the sum of all interior angles.



Design

Your teacher will group you in teams of 4 members each. Assign a letter to someone from A to D. Student A creates an illustration about congruent angles. Student B creates an illustration about complementary angles. Student C creates about supplementary angles. Student D creates about vertical angles. For your angle pair, put the measure of only one of the angles. Your classmates will solve for the value of the missing angle measure. Pass your papers clockwise. Solve for the value of the angle. Once every one is done, pass the paper to the next person. Continue until the owner has their paper back. Check the answers of your groupmates.



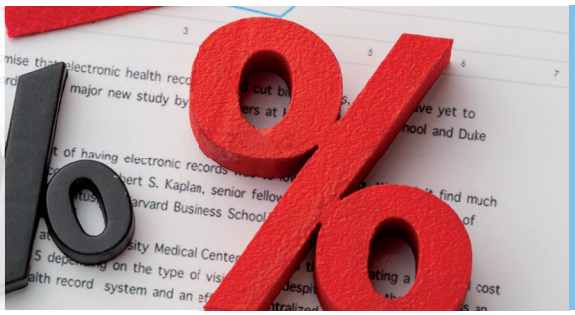
Fun Facts

Many man-made objects, from quilts to floor tiles, are adorned with polygon patterns. Artists and designers often incorporate polygons into their creations for aesthetic appeal and mathematical precision.



LESSON 5

Application of Percentages



Objectives

- Solve problems involving percentage increase and decrease.
- Solve money problems involving percentages.
- Create a financial plan.

Examine

A business buys products in bulk from a supplier overseas with a big discount. It then sells it to several retail stores with an increase for a profit. Have you ever wondered how much is the discount that companies get from buying in bulk? How about how much they increase the prices?

In this lesson, we are to solve problems involving percentage increase and decrease. We are to solve money problems such as discount, commission, sales tax, simple interest using percentage. Lastly, our goal is to help you create a financial plan.

Percentages and Rates

Application of Percentages

Use of Rates

Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).

Recall

Remember a time when your parents were excited to buy something at a great discount. What was the item they bought? How much money did they save?



Explore

When you were in grade 6, you learned about percentages and its relationships with decimals and fractions. For this lesson, we are going to learn about the application of percentages.

Here is a situation where we can apply percentages. Ryan and his cousins were planning to buy and sell school supplies just before school starts. They bought the supplies from Wholesale Center in bulk. A single notebook costs PhP 25. But if they bought a set of 100 pieces, they would only have to pay PhP 2250 instead of PhP 2500. What was the percent of the decrease in price?

The given problem involves getting the percent decrease. Percent decrease is the rate of decrease in value compared to the original value.

To find the percent decrease, Ryan must find how much was reduced from the original value or amount of 100 notebooks.

First, determine how much was deducted from the original value. Subtract PhP 2250 (wholesale price) from PhP 2500 (original price).

Amount Reduced:

$$2500 - 2250 = 250$$

Next, determine what percent of the original amount was the amount deducted.

$$\frac{250}{2500} = 0.10$$
$$= 10\%$$

Here, 250 is the percentage (P, a part) and 2500 is the base (B, a whole or total). We are looking for the rate (R, %).

Recall that to change a decimal to percent, just multiply it by 100. This is the same as moving the decimal point 2 places to the right.

The deducted amount is 10% of the original amount. Therefore, the percent decrease was 10%.

Ryan and his cousins were going to resell the spiral notebooks that they had bought for PhP 25 each. If they were to sell the notebooks for PhP 30 each, by what percentage would they increase the price?

This is an example of a problem involving percent increase. Percent increase is the ratio of the amount added or increased to the original value or amount in percent form.

To find the percent increase, first get the amount that is added from the original price of a notebook. Subtract PhP 25 (original price) from PhP 30 (new price).

$$\text{Amount Added} = 30 - 25 = 5$$

Then, determine what percent of the original price is the amount added.

$$\frac{5}{25} = 0.2 \text{ or } 20\%$$

The original price of a notebook was increased by 20%. Therefore, the percent increase was 20%.

There are many practical applications of percent increase and decrease. First, we have the discount.

Ryan saw bond papers on sale at 20% off. One ream of short bond paper costs PhP 320. How much was the discount? This is an example of a discount problem. Discount problems are examples of percent decrease problems.

Recall: Percentage (P, a part) = Base (B, a whole or total) × Rate (R, %)

Let us identify the given values first.

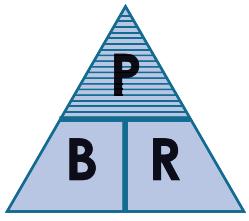
Discount	Original Price	Rate of Discount
?	320	20%

Since getting the discount is an application of percent, we can identify the percentage, rate, and base in this problem.

Percentage (P)	Base (B)	Rate (R)
Discount	Original Price	Rate of Discount
?	320	20%

The discount is the percentage, the original price is the base and the rate of discount is the rate. Therefore, we can find the discount by multiplying the rate of discount by the original price.

$$P = B \times R$$
$$\text{Discount} = 320 \times 20\%$$
$$= 320 \times 0.2$$
$$= 64$$



If Ryan bought a ream of short bond paper at 20% discount, he would be able to save PhP 64.

Discount is only one of the many applications of percent. Other common applications are commission, interest, and sales tax.

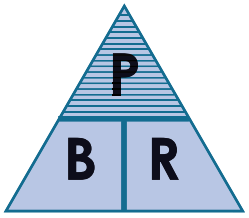
Let's tackle them one by one so that Ryan and his cousins can prepare to be business people.

First, commission is the amount paid for a service provided. It is a percentage of the sales.

To compute for the commission, you need the rate of commission and the total sales. Since commission is an application of percent, we will make use of the percentage, rate, and base.

Percentage (P)	Base (B)	Rate (R)
Commission	Total Sales	Rate of Commission

To get the commission:



Percentage = Rate × Base

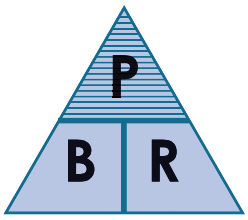
Commission = Total Sales × Rate of Commission

If you get 5% commission for every PhP 20 000 total sales, how much is the commission?

Summarize the given information in a table.

Percentage (P)	Base (B)	Rate (R)
Commission	Total Sales	Rate of Commission
?	20 000	5%

Express the rate of commission in decimal form.



Percentage = Base × Rate

Commission = 20 000 × 5%

Commission = 20 000 × 0.05 = 1000

Therefore, the commission is PhP 1000.

Next, tax is an amount collected by the government based on goods, properties, and salaries.

To compute for the sales tax, you need the tax rate and the price or cost of the goods or services.

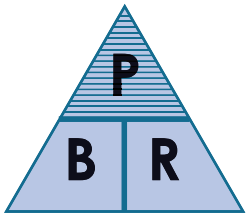
Percentage (P)	Base (B)	Rate (R)
Tax	Cost or Salary	Rate of Tax

How much is the tax if the tax rate is 12% and the price of a pair of sneakers exclusive of tax is PhP 2500? What is the cost of the sneakers inclusive of tax?

Summarize the given information in a table.

Percentage (P)	Base (B)	Rate (R)
Tax	Cost or Salary	Rate of Tax
?	2500	12%

Express the rate of tax as a decimal.



Percentage = Base × Rate

Tax = 2500 × 12%

Tax = 2500 × 0.12

Tax = 300

The tax is PhP 300. This means that PhP 300 is added to the price of the sneakers.

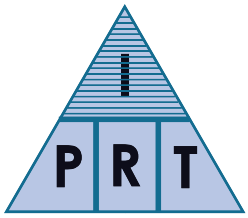
Therefore, the sneakers cost PhP 2800 inclusive of tax.

Last, interest is the amount paid for the use of money. It is a percentage of the principal amount.

To compute for the simple interest, you need the rate of interest, the principal or the amount borrowed, and the time or period of borrowing. Like commission, interest is an application of percent but with inclusion of time.

Percentage	Base	Rate	Time
Interest	Principal	Rate of Interest	Period of Borrowing

To get the interest (I), we will use the formula:



Interest = Principal × Rate × Time

I = P × R × T

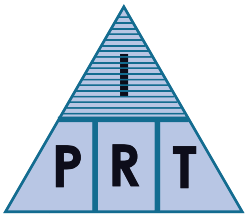
If PhP 40 000 was deposited in a bank at an annual interest rate of 10%, how much would you get after 2 years?

Summarize the given data in a table.

Interest (I)	Principal (P)	Rate of Interest (R)	Time (T)
?	40 000	10%	2 years

The term annual or per annum means "per year" or "yearly."

Express 10% as 0.10.



I = P × R × T

I = 40 000 × 10% × 2

I = 40 000 × 0.10 × 2

I = 8000

The interest after two years is PhP 8000. Adding the interest to the principal amount, we get PhP 48 000.

Thus, you would get PhP 48 000 after 2 years.

We have been discussing about money matters. Money is a medium of exchange, a measure of value, and a way to save.

Here are some ways that different people perceive money.

Many people view money as a source of security and stability. It represents the ability to cover basic needs, such as food, shelter, and healthcare.

Money can be seen as a symbol of power, influence, or status in society. Having more money can mean having more influence or a higher social standing.

For some, money symbolizes freedom—the freedom to make choices, travel, take risks, or even retire early.

Some people gauge their success, worth, or accomplishments based on their financial wealth.

Others view money is a tool to help people. With more money, they can share more to those in need.

The first step to figuring out what to do with our money is to set goals. What is the purpose of money?

As a grade 7 student, you have money because your parents give you an allowance. You can use this money for your transportation and food.

Some of you may be getting money from part-time jobs like doing household chores and tutoring younger students. These are possible sources of income.

Since you have some money, you can prepare a budget. This is a spending plan based on how much money you have and want to spend. This tracks how much money we receive (income) and how much money we spend (expenses).

You have a choice whether to spend your money on your needs or wants. There is a limit to our spending.

We do not want to borrow money as we may have trouble repaying it in the future. We are asked to control our spending. We are called to appreciate what we already have and work hard to get what we desire in the future.

This is just a short introduction on financial planning. As you get older, you will learn about the emergency fund, money for unplanned expenses. You will also learn about insurance, which is protection from financial loss.



Try This

Imagine that you were given PhP 1000. List down in detail how you would spend the money.

1. What are the things you want to buy?
2. How much do they cost?



Key Points

- Percent increase is the ratio of the amount added or increased to the original value or amount in percent form. Percent decrease is the rate of decrease in value compared to the original value.
- The concept of percentages can be applied to money-related problems, such as calculating discounts, interest rates, or tax.
- Financial responsibility is important even at a young age. Track income and expenses to stay on track with your financial plan.



Do

Read the situations and answer the following questions.

A. Ryan saw other items on sale. If he bought a pack of 200 ballpens, he would just have to pay PhP 4250 instead of PhP 5000.

1. How much was the original amount?
2. How much was the new amount?
3. How much was the amount saved?
4. What percent of the original amount was the deducted amount?
5. How much was the percent decrease?

B. Ryan bought notebooks for PhP 25 each and suggested to sell the notebook for PhP 27.50.

1. How much was the original amount?
2. How much was the amount added?
3. What was the rate of the amount added compared to the original amount?
4. How much was the percent increase?

C. Long bond papers were on sale at 15% off. How much was the discount if a ream of long bond paper originally costs PhP 365?

1. What is the original price?
2. What is the rate of discount?
3. How do you compute for the percentage?
4. How much was the discount?

Brainstorm

Your teacher will assign you a partner. You each create a print ad for any item on sale. Be creative. Give the original price and rate of discount, exchange print ads and let your partner compute for the discount.

Design

Your teacher will group you into groups of 4 students each. Form a circle. Get a piece of paper each. You will write your favorite meal and then write a price for it. Write 3 percentages and indicate if it is an increase or decrease. For example, you may write adobo for PhP 100 and below it write +10%, -10%, and +20%. Pass your papers clockwise. Solve for the new price of the meal for the first percentage. Pass the papers clockwise again. This time, solve for the new price of the meal for the second percentage. Finally, pass the papers clockwise and solve for the new price of the meal for the last percentage. Give back the papers to the owners and they will check the answers.

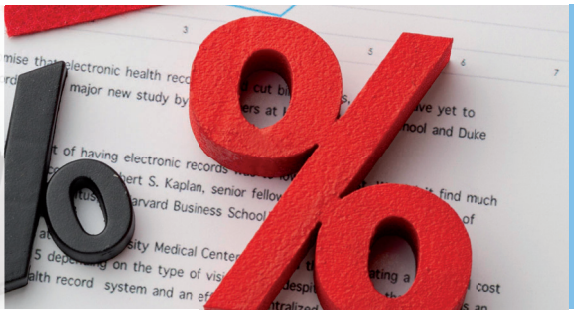
Fun Facts

The term discount possibly came from the French term descompte and the Latin term discomputus.



LESSON 6

Use of Rates



Objectives

- Identify and explain the use of rates.
- Solve problems involving rates.

Examine

Understanding rates is crucial in many situations. Have you ever compared prices in a grocery story to see which has the best value? For instance, if a 1-liter bottle of juice costs 80 pesos, and a 500 mL bottle of the same juice costs 50 pesos, you try to determine which option provides a better rate per liter.

In this lesson, we are to identify and explain the use of rates. We are going to discuss and solve problems about different kinds of rates such as unit rates, exchange rates, interest rates, and speed in real-life scenarios in the Philippine context.

Percentages and Rates

Application of Percentages

Use of Rates

Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).



Recall

Encircle the letter of the correct answer.

1. Which of the following is an example of a rate?
   a. 50 apples
   b. 5
   c. 20 kilometers per hour
   d. 2:3

2. The term "kph" stands for:
   a. Kilometers per hour
   b. Kilograms per hectare
   c. Kiloliters per house
   d. Kites per hill

3. If a car travels at a constant rate of 60 kilometers per hour, how far would it travel in one hour?
   a. 30 kilometers
   b. 45 kilometers
   c. 60 kilometers
   d. 120 kilometers

4. The rate that describes how much one quantity changes in comparison to another is called a:
   a. Ratio
   b. Percentage
   c. Unit rate
   d. Proportion

5. What does "per" usually indicate in the context of rates?
   a. Addition
   b. Subtraction
   c. Multiplication
   d. Division

6. Which unit represents speed?
   a. Kilogram
   b. Meter
   c. Second
   d. Meters per second

7. If a car travels 180 kilometers in 3 hours, what is its speed in kilometers per hour?
   a. 30 km/h
   b. 50 km/h
   c. 60 km/h
   d. 70 km/h

8. If 5 shirts cost PhP 1000, how much does each shirt cost?
   a. PhP 100
   b. PhP 200
   c. PhP 250
   d. PhP 500

9. Which of the following is the best buy based on the rate provided?
   a. 4 candies for PhP 20
   b. 10 candies for PhP 50
   c. 20 candies for PhP 110
   d. 15 candies for PhP 80

10. Which is a faster speed?
    a. 10 kilometers in 2 hours
    b. 15 kilometers in 2 hours
    c. 5 kilometers in 1 hour
    d. 20 kilometers in 5 hours



Explore

You discussed about ratios in grade 6. Today's lesson is about rates.

A rate is a special kind of ratio where the two quantities being compared have different units. It tells us how one quantity changes in comparison to another quantity over a specific period or interval.

Common examples of rates are kilometers per hour (speed) and pesos per kilogram (price of goods). In the Philippines, when you buy rice, you might see it priced as "PhP 50 per kilo." This means for every kilogram of rice, you need to pay PhP 50. The "per" often indicates a rate.

A unit rate describes how many units of the first type of quantity corresponds to one unit of the second type of quantity.

Examples of unit rates are walking 10 000 steps per day, 80 heart beats per minute, and 50 pesos per kilogram.

We can calculate the unit rates.

Here is a problem. At the palengke, Terry is selling 3 kilograms of mangosteens for PhP 480. What is the price of mangosteens per 1 kilogram?

We divide PhP 480 by 3 kg to get the price per 1 kg.

$$\frac{PhP\,480}{3\,kg} = 160\frac{PhP}{kg}$$

If we buy 1 kilogram of mangosteens, we pay Terry PhP 160.

Here is another problem. Mang Juan earns PhP 600 for 8 hours of work. How much does he earn in 1 hour?

We divide PhP 600 by 8 hours to get the price per hour.

$$\frac{PhP\,600}{3\,h} = 75\frac{PhP}{h}$$

Mang Juan earns PhP 75 per hour of work.

Here is our last problem on unit rates. A jeepney consumes 10 liters of diesel for every 55 kilometers. How much distance can it travel per liter?

We divide 55 km by 10 L to get the distance per liter. This is also called fuel economy.

$$\frac{55\ km}{10\ L} = 5.5\frac{km}{l}$$

The jeepney travels 5.5 km per liter.

Another type of rate is exchange rate.

An exchange rate tells us how much one currency is worth in terms of another currency. It's like a price tag that lets you know how many Philippine pesos (PhP) you need to spend or will receive when you exchange it for another country's money, like US dollars (USD) or Japanese yen (JPY). For example, if the exchange rate is PhP 57 to 1 USD, it means you need 57 Philippine pesos to get 1 US dollar. Exchange rates can change based on various factors like the economy and demand for the currency. When you travel to another country or buy things online from overseas, understanding the exchange rate helps you know how much you're spending in terms of your own money.

Let us convert our Philippine pesos to US dollars.

Your tita, who is an OFW, gave you 86 US dollars for your birthday. How much is this in Philippine pesos?

The current exchange rate is 57 pesos = 1 US dollar. Since we have 86 US dollars, we multiply 86 dollars by 57 pesos per 1 US dollar.

$$86\,USD\left(\frac{57\,PhP}{1\,USD}\right) = PhP\,492$$

Note that we cancel the USD unit from the first number with the USD unit from the denominator of the second number. We are left with the unit of PhP.

You received PhP 4902 for your birthday.

Let us now do the reverse. Let us convert Philippine pesos to US dollars.

Your tito is going to the US. He goes to the bank to exchange PhP 15 495 to US dollars. The exchange rate is 57 pesos = 1 US dollar. How much US dollars will he receive?

Since we have PhP 15 495, we divide this by PhP 57. We invert and use the rate of 1 USD = PhP 57.

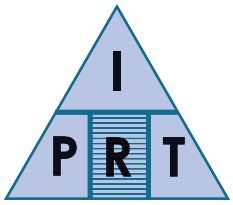
$$PhP\,15\,495\left(\frac{1\,USD}{57\,PhP}\right) = 271.84\,USD$$

We cancel the PhP unit from the first number with the PhP unit from the denominator of the second number. We are left with the unit of USD.

The answer is rounded to 2 decimal places. Your tito will get USD 271.84.

Another kind of rate is interest rate. Banks pay interest on savings account to encourage its customers to put their money in the bank. In the previous lesson, we discussed about solving for the interest. Let us have an example on solving for the interest rate.

To compute for the rate of interest (R), we use the formula:



$$\text{Rate of Interest} = \frac{\text{Interest}}{\text{Principal} \times \text{Time}}$$

$$R = \frac{I}{P \times T}$$

What is the rate of interest per annum if the principal is PhP 36 000 and the interest is PhP 5400 for 3 years?

Summarize the given data in a table.

Interest (I)	Rate of Interest (R)	Principal (P)	Time (T)
5400	?	36 000	3

$$R = \frac{5400}{36\,000 \times 3}$$

$$R = \frac{5400}{108\,000}$$

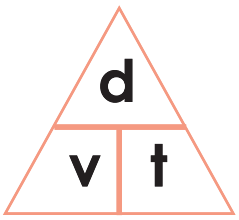
$$R = 0.05$$

$$R = 5\%$$

Therefore, the rate of interest is 5%.

The last topic on rates is on speed, distance, and time.

Speed refers to how fast an object is moving. It is computed by determining how much distance is covered over a specified time. Speed is represented by the letter v. Here are the formulas for speed, distance, and time.



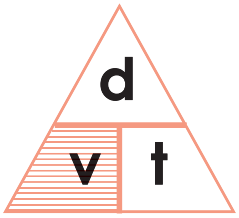
$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Let's solve for the average speed of the car that covered a distance of 160 km in 2 hours. We just use the formula for speed.

The average speed is the total distance traveled divided by the total time elapsed.



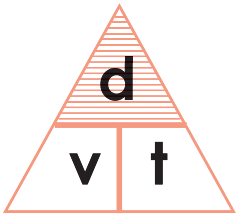
$$v = \frac{d}{t}$$

$$v = \frac{160\ km}{2\ h}$$

$$v = 80\ km/h$$

The average speed of the car is 80 kilometers per hour or 80 km/h.

If the man continued to drive for another two hours at 90 km/h, what would be the total distance traveled? This time, we use the formula for distance.



$$d = v \times t$$

$$d = 90\ km/h \times 2\ h$$

$$d = 180\ km$$

Therefore, the car would have traveled 180 km farther. To get the total distance traveled, we just add it to the previous distance.

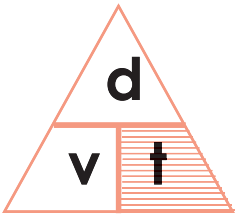
$$\text{Total distance} = 160\ km + 180\ km = 340\ km$$

Thus, the car would cover a total distance of 340 km.

If the man had to drive a total of 490 km to reach his destination and continued at the speed of 100 km/h, how many more hours would he travel?

To solve the problem, we will use the formula for time. Note that we first have to compute for the distance he would still have to travel.

We subtract 340 km from 490 km to get the remaining distance.



$$t = \frac{d}{v}$$

$$t = \frac{490\ km - 340\ km}{100\ km/h}$$

$$t = \frac{150\ km}{100\ km/h}$$

$$t = 1.5\ hours$$

Hence, the man would have to drive for another 1.5 hours.

Determining which two variables are given and which variable is missing can help in solving speed problems.



Try This

Solve this problem. Please show your step-by-step solution. A man drove 360 km for 4.5 hours. What was his average speed?

1. What is the formula for speed?
2. What is the average speed?

You have learned much about rates. There are many more. In science, you will learn about the relationship among force, mass, and acceleration. In math grade 9, you will continue to learn more about them as the slope of a line is an example of a rate of change.



Key Points

- A rate is a special ratio that compares two different kinds of numbers, such as kilometers per hour or pesos per kilogram. It tells us how much of one thing there is for every unit of another thing.
- Rates are everywhere around us, from shopping to traveling. They help us compare different quantities and make informed decisions. By understanding and being able to calculate rates, we can solve real-world problems effectively.



Do

Encircle the letter of the correct answer.

1. What is a rate?
   a. A type of fraction
   b. A comparison of two numbers of the same kind
   c. A comparison of two numbers of different kinds
   d. A type of decimal

2. Which of the following is a rate?
   a. 5 apples
   b. 50 pesos per kilogram
   c. 30 boys
   d. 15 minutes

3. How do you calculate the unit rate?
   a. Multiply the two numbers
   b. Add the two numbers
   c. Divide the first number by the second number
   d. Subtract the two numbers

4. What does the rate "20 kilometers per hour" describe?
   a. The distance traveled in 20 hours
   b. The speed of an object
   c. The cost of traveling 20 kilometers
   d. The time it takes to travel 20 kilometers

5. Which of the following rates describes a speed?
   a. 20 pesos per candy
   b. 5 books per shelf
   c. 45 kilometers per hour
   d. 3 cups of flour per recipe

6. Which rate represents the cost of an item?
   a. 15 minutes per task
   b. 5 liters per bottle
   c. 120 pesos per toy
   d. 50 kilometers per hour

7. If a car travels 60 kilometers in 2 hours, what is its speed?
   a. 120 kilometers per hour
   b. 30 kilometers per hour
   c. 2 kilometers per hour
   d. 60 kilometers per hour

8. If 4 shirts cost 800 pesos, how much does 1 shirt cost?
   a. 100 pesos
   b. 200 pesos
   c. 300 pesos
   d. 400 pesos

9. Shiela and Johans took two different routes to travel from City A to City B. Shiela traveled 200 kilometers in 4 hours, while Johans traveled 250 kilometers in 5 hours. Who had a faster rate of travel and by how much?
   a. Shiela by 5 kilometers per hour
   b. Johans by 5 kilometers per hour
   c. Both traveled at the same rate
   d. Johans by 10 kilometers per hour

10. A company is looking at the efficiency of two machines. Machine X produces 20 units per hour using 10 watts of power, while Machine Y produces 15 units per hour using 5 watts of power. Which machine has a better rate of production per watt?
    a. Machine X
    b. Machine Y
    c. Both machines have the same rate of production per watt
    d. Cannot be determined from the information given



Brainstorm

Your teacher will assign you a partner. Assign one person to be Person A and the other to be Person B. Person A creates a problem about exchange rate. Give the exchange rate of any currency and Philippine pesos. It can just be an estimate. Write a problem about converting money in a foreign currency to Philippine pesos. Person B creates a problem about interest rate. Imagine that you are a bank manager. Write a problem about finding the interest rate given the principal, interest, and time of a customer deposit. Exchange papers and solve for the amount in PhP or the interest rate. Once both of you are done, present to one another your solutions.



Design

Your teacher will group you into groups of 4 students each. Form a circle. Get a piece of paper each. You will write your 3 favorite fruits and then write rates for it. An example is rambutan costs PhP 160 for 4 kilograms. If you don't know the exact price, it is okay. Make up your own prices for this activity. Pass your papers clockwise. Solve for the unit rate of the first fruit. Pass the papers clockwise again. This time, solve for the unit rate of the second fruit. Finally, pass the papers clockwise and solve for the unit rate of the last fruit. Give back the papers to the owners and they will check the answers.



Fun Facts

Usain Bolt, the Jamaican sprinter, set a world record by running 100 meters in 9.58 seconds in 2009.

LESSON 7

Rational Numbers



Objectives

- Describe given rational numbers as fractions, decimals, or percentages.
- Order rational numbers on a number line.
- Perform operations on rational numbers.

Examine

Rational numbers are used in many situations. Cooking involves using rational numbers when measuring ingredients. Recipes often require fractional amounts such as one-half cup of sugar which is an example of rational numbers. How have you used rational numbers in cooking?

Managing finances involves dealing with rational numbers. For instance, when budgeting for expenses, calculating amounts spent or saved, or dividing money among friends or family members, rational numbers are used. How have you used rational numbers with money?

Rational Numbers

Rational Numbers

Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).

While traveling or commuting, rational numbers come into play when measuring distances and speeds. We just discussed these in the previous lesson. When measuring objects or heights, rational numbers are used. Lastly, scores and statistics in sports often involve rational numbers. Where else have you seen the use of rational numbers?

In this lesson, we are to express and convert rational numbers as fractions, decimals, or percentages and vice versa. We are going to plot and arrange rational numbers on a number line. Our goal is to help you perform the operations of addition, subtraction, multiplication, and division on rational numbers.



Recall

Encircle the letter of the correct answer.

1. What is the decimal representation of $\frac{1}{4}$?
   a. 0.14 b. 2.5 c. 0.4 d. 0.25

2. Which of the following is equivalent to 50%?
   a. $\frac{1}{4}$ b. $\frac{1}{2}$ c. $\frac{3}{4}$ d. $\frac{1}{3}$

3. What is the percentage representation of 0.75?
   a. 75% b. 50% c. 25% d. 60%

4. Which of the following is a rational number?
   a. $\sqrt{3}$ b. π c. 1.5 d. $\sqrt{-4}$

5. On a number line, which value is greater, 2.5 or 1.5?
   a. 2.5 c. Both are equal
   b. 1.5 d. It cannot be determined

6. Which is the smallest fraction?
   a. $\frac{1}{2}$ b. $\frac{3}{4}$ c. $\frac{2}{3}$ d. $\frac{1}{3}$

7. Which of the following is the result of subtracting 2.5 from 5.5?
   a. 2 b. 3 c. 3.5 d. 8

8. If $\frac{5}{6}$ is subtracted from $\frac{6}{6}$, what will be the result?
   a. $\frac{1}{6}$ b. 0 c. $\frac{1}{5}$ d. $\frac{2}{12}$

9. Which operation will convert a fraction to a decimal?
   a. Addition b. Subtraction c. Multiplication d. Division

10. Between which two whole numbers does $\frac{5}{7}$ lie?
    a. 0 and 1 b. 1 and 2 c. 5 and 7 d. 6 and 7



Explore

A rational number is any number that can be written as a fraction, where both the numerator and the denominator are integers, provided that the denominator is not equal to zero. An integer is a whole number (not a fractional number) that can be positive, negative, or zero.

Basically, all integers are rational numbers and all the numbers in between two consecutive integers are also rational numbers.

In general, rational numbers are the quotient of two integers. Rational numbers can be written in fraction, decimal, or percentage notations. Here is a short review of how to convert fractions to decimals and vice versa which you learned in grade 4. After, how to convert percentages to fractions or decimals and vice versa is briefly discussed. Their relationships were discussed in grade 6.

In order for you to convert a rational number in fraction form into its corresponding decimal form, you only need to divide the numerator by the denominator. For example, to convert $\frac{1}{2}$ into decimal form, divide 1 by 2 to get 0.5.

On the other hand, in order to convert decimals into their corresponding fraction forms, we just read it the way it should be read then simplify if possible. For example, 0.75 is read as seventy-five hundredths which is equal to $\frac{75}{100}$ in fraction form. This is simplified to $\frac{3}{4}$ because 75 and 100 are divided by their GCF which is 25.

To convert decimals to percentages, multiply the decimal by 100 and then put the % sign. In our previous example, 0.75 times 100 = 75. We then add the % sign at the end to get 75%.

To convert percentages to decimals, divide the percentage by 100 and remove the % sign. For example, 80% ÷ 100 = 0.8.

To convert fractions to percentages, first convert it to a decimal and then multiply it by 100 and add the % sign. From our earlier example, $\frac{1}{2}$ is 0.5 as a decimal. We multiply 0.5 by 100 to get 50 and then put the % sign at the end to get 50%.

To convert percentages to fractions, first convert it to a decimal. Then, convert it to a fraction. For example, 40% = 0.4. 0.4 in fraction form is $\frac{4}{10}$. We simplify it to $\frac{2}{5}$.

Take note that fractions can be proper fractions, improper fractions, and mixed numbers. These were discussed back in grade 4. For decimals, there are two types that are rational numbers. They are the finite or terminating decimals and non-terminating, repeating decimals.

Finite or terminating decimals are those numbers which come to an end after a few repetitions after the decimal point. 0.5, 2.456, 123.456, etc. are all examples of terminating decimals. This was discussed back in grade 5. Remember that a rational number with a finite decimal part or for which the long division terminates (stops) after a definite number of steps.

Non-terminating, repeating decimals go on forever but have a repeating pattern. This was discussed back in grade 6. For example, $\frac{1}{3}$ is 0.333... (the 3 repeats forever).

Rational numbers can be easily be represented on a number line by just following some simple steps. You have learned how to do this for fractions and decimals when you were in grade 4. But before going to the number line, don't forget to check for the negative and positive sign of the rational number. Positive rational numbers are always represented on the right side of the zero on the number line. While negative rational numbers are always represented on the left side of zero on the number line.

Let us show 20%, -1.4, and $-\frac{9}{5}$ on the number line.

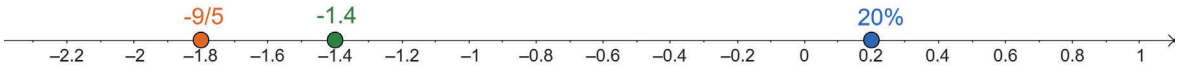
Before we plot the points, to make it simple, let us convert the numbers to decimals.

20% = 0.2

-1.4 is already a decimal.

$-\frac{9}{5}$ = -1.8

Here are the 3 points on the number line.



The order of the rational numbers from smallest to largest is $\frac{-9}{5}$, -1.4, and 20%.

In general, if you want to arrange rational numbers on a number line, you may determine in which consecutive integers they are in between and try to locate it. Remember that the number on the left on a number line is smaller than the right.

The last topic is on the operations on rational numbers. Most of the time, you will be dealing with fractions and decimals. To make things simpler, convert all the rational numbers to either fractions or decimals before performing the operation.

$$\begin{array}{r} {\scriptstyle 1\ 1\ 1} \\ 14.78 \\ +\ 45.39 \\ \hline 60.17 \end{array} \qquad \begin{array}{r} {\scriptstyle 3\ 10\ 10\ 9\ 10} \\ 41.080 \\ -\ 39.197 \\ \hline 1.883 \end{array}$$

In adding or subtracting decimals, arrange the decimal numbers in a column such that the decimal points are aligned, then add or subtract just like with whole numbers. You discussed this back in grade 5. Here are some examples.

$$\begin{array}{r} 75 \\ \times\ 0.5 \\ \hline 37.5 \end{array}$$

In multiplying decimals, just multiply the as if you are multiplying whole numbers. Starting from the rightmost end of the product, move the decimal point to the left with the same number of places as the sum of the decimal places in the multiplicand and multiplier. You also discussed this back in grade 5. Here is an example.

$$\begin{array}{r} 19.25 \\ 25\overline{)\,481.25} \\ -\ 25 \\ \hline 231 \\ -\ 225 \\ \hline 62 \\ -\ 50 \\ \hline 125 \\ -\ 125 \\ \hline 0 \end{array}$$

In dividing decimals, if the divisor is a whole number, divide the dividend by the divisor applying the rules for whole numbers. The position of the decimal point is the same as that in the dividend. If the divisor is not a whole number, make the divisor a whole number by moving its decimal point to the rightmost end, move the decimal point of the dividend to the right with the same number of places as we did in the divisor. You may now perform the division. Again, you discussed this back in grade 5. Here is an example.

Here are the steps on how to perform operations with fractions.

First, here is how to add or subtract fractions. If two fractions have the same denominator, simply add or subtract their numerators. You discussed this back in grade 3. For instance, $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$.

If they have different denominators, find a common denominator first. Once, they have the same denominator, you may add or subtract. You discussed this back in grade 4. For example, to subtract $\frac{1}{4}$ from $\frac{2}{3}$, the least common denominator is 12. Convert each fraction so they have this new denominator and then subtract: $\frac{8}{12} - \frac{3}{12} = \frac{5}{12}$.

Multiplying fractions is straightforward. Multiply the numerators with each other and the denominators with each other. You discussed this back in grade 5. For instance, $\frac{2}{3}\left(\frac{1}{4}\right) = \frac{2}{12} = \frac{1}{6}$.

To divide rational numbers, multiply by the reciprocal of the divisor. The reciprocal of a number is obtained by switching its numerator and denominator. You also discussed this back in grade 5. For example, $\frac{3}{4} \div \frac{2}{5} = \frac{3}{4}\left(\frac{5}{2}\right) = \frac{15}{8}$.



Try This

Solve these 2 problems. Please show your step-by-step solution. Take note of the LCD for subtraction and the reciprocal for division.

1. $\frac{7}{9} - \frac{2}{3}$

2. $\frac{3}{4} \div \frac{2}{3}$



Key Points

- Rational numbers are numbers that can be written as a fraction where both the numerator (the top number) and the denominator (the bottom number) are whole numbers, and the denominator is not zero. It can be described as a fraction, decimal, or percentage.
- Rational numbers can be placed on a number line. Numbers to the left are smaller, and numbers to the right are bigger.
- Here is how to perform operations on rational numbers. Addition and subtraction are just like how you add or subtract whole numbers, but sometimes you need a common denominator for fractions. To perform multiplication, multiply the numerators (top numbers) together and then the denominators (bottom numbers) together. For division, flip the second fraction upside-down (this is now its reciprocal), then multiply.



Do

I. Encircle the letter of the correct answer.

1. Which of the following is a rational number?

   a. $\sqrt{2}$ b. π c. 1.75 d. e

2. What is the decimal representation of $\frac{1}{2}$?

a. 0.2 b. 0.5 c. 0.02 d. 1.2

3. Which of the following is equal to 45%?

a. 0.45 b. $\frac{45}{100}$ c. Both A and B d. Neither A nor B

4. Which option correctly describes 0.75 as a fraction?

a. $\frac{1}{2}$ b. $\frac{3}{5}$ c. $\frac{3}{4}$ d. $\frac{4}{5}$

5. The number $\frac{3}{5}$ is the same as:

a. 0.6 b. 60 c. 60% d. 0.06

6. 0.75 can also be represented as:

a. 75% b. 7.5% c. $\frac{3}{4}$ d. Both A and C

7. If Maria spent 25% of her money, which is equivalent to $\frac{1}{4}$, how would you write 25% as a decimal?

a. 0.025 b. 0.25 c. 2.5 d. 25

8. If you add $\frac{2}{3}$ and $\frac{1}{6}$, what do you get?

a. $\frac{1}{2}$ b. $\frac{5}{6}$ c. $\frac{3}{9}$ d. $\frac{3}{8}$

9. Which of the following numbers is the smallest?

a. $-\frac{1}{2}$ b. 0.5 c. -0.49 d. 0

10. Arrange these numbers from least to greatest: $\frac{2}{3}$, 0.66, $\frac{3}{5}$

a. $\frac{2}{3}$, 0.66, $\frac{3}{5}$ b. 0.66, $\frac{3}{5}$, $\frac{2}{3}$ c. $\frac{3}{5}$, 0.66, $\frac{2}{3}$ d. 0.66, $\frac{2}{3}$, $\frac{3}{5}$

II. TRUE or FALSE: Write the T is the statement is true. Otherwise, write F.

_____1. Any fraction can be converted to decimal.

_____2. There are 9 rational numbers in between 0 and 1.

_____3. Zero is a rational number.

_____4. Any decimal number can be converted into fraction.

III. Draw a number line to compare the 2 rational numbers.

1. 2 and -4
2. $\frac{1}{3}$ and $\frac{5}{3}$

IV. Perform the operations. Reduce to lowest terms.

1. $\frac{12}{17} + \frac{3}{17} =$
2. $\frac{7}{10} + \frac{2}{1} + \frac{8}{10} =$
3. $\frac{1}{2} + \frac{2}{3} =$
4. $3\frac{7}{8} + 1\frac{5}{8} =$
5. $6\frac{3}{10} + 5\frac{10}{100} =$
6. $\frac{8}{11} - \frac{5}{11} =$
7. $\frac{7}{16} - \frac{5}{16} =$
8. $\frac{7}{9} - \frac{2}{3} =$
9. $10 - 3\frac{2}{3} =$
10. $5\frac{3}{8} + 1\frac{1}{6} =$
11. $\frac{-18}{54}\left(\frac{6}{7}\right) =$
12. $\frac{15}{4} \div \frac{-23}{5} =$



Brainstorm

Your teacher will assign you a partner. Use the same rational numbers as the example before. Work together to plot 20%, -1.4, and $-\frac{9}{5}$ on the number line but this time, convert them first to fractions. After doing this activity, discuss if it is easier to use fractions or decimals for a number line.



Design

Your teacher will group you into groups of 3 students each. Form a circle. Assign people to be Person A, Person B, and Person C. Given the fractions, convert them first to decimals and then to percentage. After, explain to your groupmates how you converted them. Finally, discuss which among converting proper fractions, improper fractions, and mixed numbers to decimals is the most difficult.

Person A: $2\frac{2}{3}$ Person B: $\frac{15}{8}$ Person C: $\frac{5}{32}$



Fun Facts

The ancient Greek mathematician Pythagoras believed that all numbers were rational, but one of his students Hippasus proved (using geometry, it is thought) that you could not write the square root of 2 as a fraction, and so it was irrational. But followers of Pythagoras could not accept the existence of irrational numbers, and it is said that Hippasus was drowned at sea as a punishment from the gods!

Unit Performance Task: Polygon Portfolio and Financial Workbook

Your teacher will group you into teams of 4 members each. Work together to do the following.

Component 1: Polygon Creation and Analysis

Each group will create a polygon portfolio featuring detailed drawings of regular and irregular polygons (5, 6, 8, 10 sides) using rulers and protractors.

For each polygon, provide a classification (regular/irregular, convex/non-convex), a description of angle pairs, and calculations of interior and exterior angles.

Present a mini-research on 4 real-world applications of polygons.

Component 2: Financial Planning and Calculations

Develop a financial workbook that includes 4 problems and solutions for percentage increase/decrease, money problems involving percentages (discounts, commissions, taxes, interest), and rate calculations.

Create a 4-panel comic strip using the skills you learned in Filipino class about 2 classmates talking about financial planning.

Include 4 rational number operations and representations in different forms (fractions, decimals, percentages).

Demonstrate understanding by explaining the processes and reasoning behind each solution.

Class Presentation

Groups will present their polygon portfolio and financial workbook to the class, explaining their processes, findings, and real-world implications.

Each presentation should be 10-15 minutes, including a Q&A session.

Rubric

Criteria	Excellent (4)	Proficient (3)	Developing (2)	Beginning (1)
Polygon Drawing & Description	Accurate, detailed drawings with comprehensive descriptions and classifications.	Mostly accurate drawings with clear descriptions and classifications.	Drawings have minor inaccuracies; descriptions and classifications are somewhat unclear.	Drawings are inaccurate; descriptions and classifications are lacking or incorrect.
Angle Analysis & Calculation	Precise calculations and insightful analysis of angle relationships.	Mostly accurate calculations; good understanding of angle relationships.	Some calculation errors; basic understanding of angle relationships.	Incorrect calculations; poor understanding of angle relationships.
Financial Problem Solving	Correct solutions with clear, logical reasoning for all financial problems.	Mostly correct solutions with reasonable explanations.	Some correct solutions; explanations lack clarity.	Incorrect solutions; poor or no explanations.
Comic Strip	Accurate and comprehensive understanding of financial planning. Well-organized with clear and logical flow.	Mostly accurate understanding of financial planning. Organized with a fairly clear flow.	Basic understanding of financial planning. Somewhat organized with a confusing or disjointed flow.	Lack of understanding of financial planning. Poorly organized with a lack of coherent flow.
Presentation & Explanation	Clear, engaging presentation with thorough explanations and confident responses to questions.	Good presentation; mostly clear explanations and good responses to questions.	Adequate presentation; some unclear explanations and hesitant responses.	Poor presentation; unclear explanations and weak responses to questions.

UNIT 2

Welcome back to the fascinating world of Mathematics!

In this unit, you'll venture further into the realm of numbers and measurements. You'll delve into topics such as square roots and cube roots, distinguishing between rational and irrational numbers, exploring various units of measurement, and discovering the concepts of volume related to cylinders and pyramids. Additionally, the unit introduces sets and explores the vastness of real numbers. The knowledge gained here will form a crucial foundation for advanced mathematical concepts. Not only will it enhance understanding in higher-level math, but it will also have numerous practical applications in everyday scenarios. Let's keep going, Grade 7 math enthusiasts!



LESSON 1

Square Roots and Cube Roots





Objectives

- Determine the square roots of perfect squares and the cube roots of perfect cubes.



Examine

Understanding square roots and cube roots is essential. We can determine the area of a square garden or field using square roots. Cube roots are useful when measuring the volume of a three-dimensional object.

Architects and builders use square roots and cube roots when calculating measurements for structures. When determining the diagonal length of a square room or the side length of a cube-shaped building, square roots and cube roots are essential.

Understanding square roots is beneficial when dealing with financial concepts like compound interest or calculating the principal amount needed to achieve a certain future value. Cube roots can also be relevant in determining investment returns over time.

Square roots and cube roots are used in various technological applications, such as image processing, computer graphics, and signal processing. Understanding these roots aids in optimizing algorithms and computations in technology-related fields. Which field are you interested in?

Irrational Numbers

Square Roots and Cube Roots

Irrational Numbers



Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).

In this lesson, we will explore the concept of perfect squares and perfect cubes. We will also understand how to determine their roots, as well as practice finding square roots of perfect squares and cube roots of perfect cubes.



Recall

Encircle the letter of the correct answer.

1. Which of the following is a perfect square?

a. 2 b. 5 c. 7 d. 9

2. Which of the following is a perfect cube?

a. 12 b. 27 c. 16 d. 8

3. Which of the following is a perfect square?

a. 12 b. 16 c. 21 d. 27

4. Calculate the value of $\sqrt{9}$.

a. 2 b. 3 c. 4 d. 5

5. If you multiply a number by itself twice, what do you get?

a. A perfect cube
b. A perfect square
c. An imperfect square
d. A fraction

6. If a number is a perfect cube, what does that mean?

a. Its cube root is a whole number
b. Its cube root is a fraction
c. Its cube root is a decimal
d. It has no cube root

7. Find the square root of 36.

a. 4 b. 5 c. 6 d. 7

8. Find the cube root of 8.

a. 2 b. 3 c. 4 d. 5

9. If $y^3 = 125$, what is the value of y?

a. 2 b. 3 c. 4 d. 5

10. Which of the following is true for a perfect square?

a. It cannot be a negative number
b. Its square root is an integer
c. Its square root is an irrational number
d. It can only be an even number



Explore

Look at the numbers below. What can you observe with these numbers? Can you identify any similarities or patterns among them?

1	4	9	16	25

These are called perfect squares. Perfect squares are whole numbers that can be expressed as the result of multiplying an integer by itself. For example, 1 is a perfect square because it can be represented as 1(1), or 1^2. 4 is also a perfect square because it can be represented as 2(2), or 2^2.

On the other hand, the opposite or inverse operation of squaring which is called the square root. When we take the square root of a number, we are finding another number that, when multiplied by itself, will be equal to the original number. For instance, the square root of 9 is 3 because 3(3), or 3^2, is equal to 9. The same also goes for the square root of 16 which is 4 since 4(4), or 4^2, is equal to 16.

The square root of a number is represented using the radical symbol, $\sqrt{}$. For example, the square root of 25 is written as $\sqrt{25}$.

Apart from perfect squares, we also have what we call perfect cubes. By looking at the numbers below, can you guess what perfect cubes are?

1	8	27

These are numbers that result from cubing an integer. For example, 1 is a perfect cube because it can be represented as 1(1)(1), or 1^3. Similarly, 8 is also a perfect cube as it can be represented as 2(2)(2), or 2^3.

Cubing also has an inverse operation called cube roots. When you take the cube root of a number, you're asking, "What number, when multiplied by itself two times, gives us the original number?" For example, the cube root of 27 is 3 because 3(3)(3), 3^3, is equal to 27.

What do you think is the symbol for cube roots?

Majority of you might have guessed the correct symbol used to represent cube roots. We write it as $\sqrt[3]{}$. For example, $\sqrt[3]{64}$ represents the number that, when cubed, equals 64. In this case, $\sqrt[3]{64} = 4$ because 4(4)(4), or 4^3, is equal to 64.



Try This

Now that you have mastered the concept of perfect cubes and cube roots, use the cube root symbol to express the following values and calculate their respective roots. What numbers do you multiply by itself 2 times to get the numbers below?

1. 216
2. 125
3. 343



Key Points

- Perfect squares are numbers that result from multiplying an integer by itself. These are called "perfect" because they have integers as square roots.
- Meanwhile, perfect cubes are numbers that result from multiplying an integer by itself twice. These are called "perfect" because they have integers as cube roots.
- Square roots, and cube roots can be represented by using the symbols $\sqrt{\ }$, and $\sqrt[3]{\ }$, respectively.



Do

I. Encircle the letter of the correct answer.

1. Which of the following numbers is a perfect square?

   a. 12 b. 49 c. 27 d. 15

2. Which of the following numbers is NOT a perfect cube?

   a. 8 b. 16 c. 27 d. 64

3. Which of these is NOT a perfect square?

   a. 25 b. 82 c. 64 d. 36

4. If a number is a perfect square, what can you say about its square root?

   a. Its square root is not a whole number
   b. Its square root is a fraction
   c. Its square root is an integer
   d. Its square root is a negative number

5. Calculate the value of $\sqrt{100}$.

a. 12 b. 11 c. 10 d. 9

6. What is $\sqrt{49}$?

a. 4 b. 5 c. 6 d. 7

7. Find the square root of 81.

a. 9 b. 8 c. 7 d. 6

8. Find the cube root of 125.

a. 2 b. 3 c. 4 d. 5

9. If $x^2 = 16$, what is the value of x?

a. 2 b. 3 c. 4 d. 5

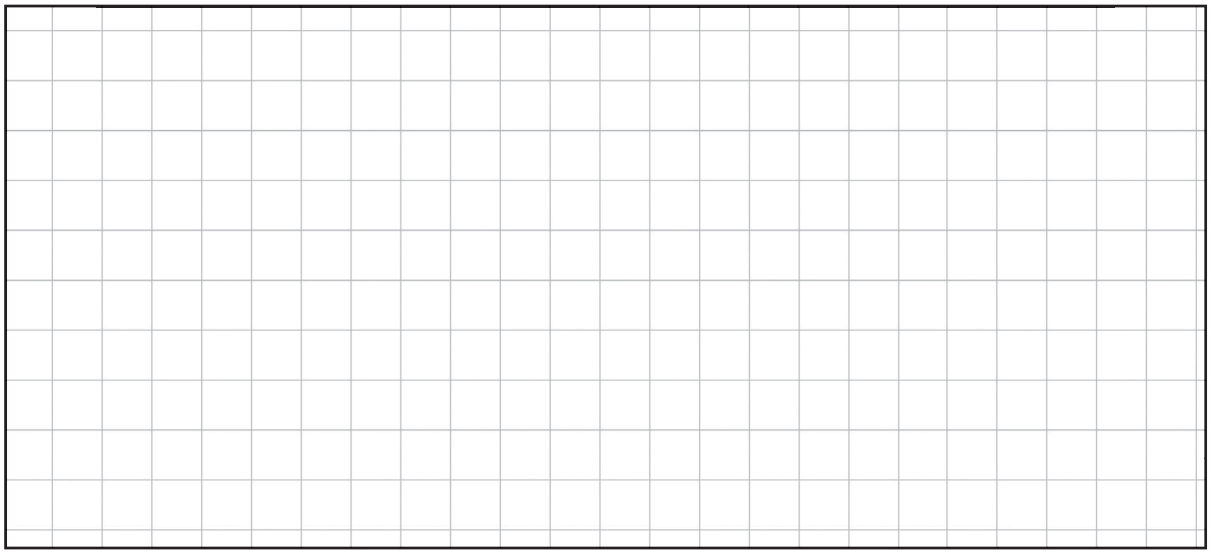
10. Which of the following is true for a perfect cube?

a. It can only be an even number
b. Its square root is an irrational number
c. Its cube root is an integer
d. It can only be a negative number

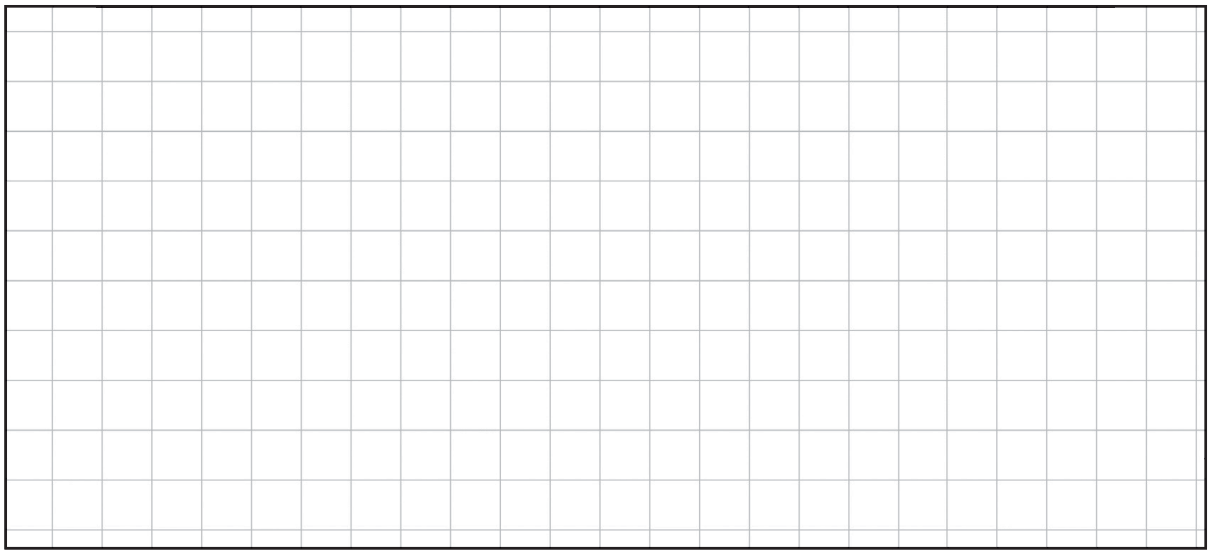
II. Complete the table of perfect squares and perfect cubes.

Number	Perfect Square	Perfect Cube
1	1	1
2		
3		
4		
5		
6		
7		
8		
9		
10		

III. Draw squares representing perfect squares and then shade in the square root.



Do the same with perfect cubes and cube roots.





Brainstorm

Your teacher will assign you a partner. Use grids or graphing paper. Work together to create pixel art using square roots or cube roots. Be creative.



Design

Your teacher will group you into teams of 4 members each. Each group brings building blocks or cubes of the same sizes. Build structures with volumes equal to the cube of the numbers 1, 2, 3, and 4 if possible.



Fun Facts

The concept of square roots can be traced back to ancient civilizations like the Babylonians and Egyptians, who used geometric methods to find areas of squares. Cube roots have also been used for millennia in architectural and engineering applications by ancient civilizations like the Egyptians and Greeks.

Some square roots, like the square root of 2, are irrational numbers, meaning they cannot be expressed as simple fractions. This fact fascinated mathematicians in ancient Greece, leading to the discovery of irrational numbers.

LESSON 2

Irrational Numbers





Objectives

- Identify irrational numbers involving square roots and cube roots.
- Place irrational numbers involving square roots and cube roots on the number line.



Examine

Have you ever wondered about π? It is an irrational number. No matter how precisely one tries to measure a circle's circumference and diameter, the exact ratio between the two is irrational.

We discussed about square roots last lesson. The square root of a number that is not a perfect square like 2, 3, and 5 is an irrational number.

When calculating the volume of certain three-dimensional objects with irregular shapes, such as spheres, cones, or pyramids, the result may involve the use of irrational numbers due to their geometric properties.

In this lesson, we will dive into the fascinating world of irrational numbers involving square roots and cube roots. We will learn what irrational numbers are, explore how to identify them when square roots and cube roots are involved, and gain the skills to place these irrational numbers on a number line.

Irrational Numbers

Square Roots and Cube Roots

Irrational Numbers



Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).



Recall

Encircle the letter of the correct answer.

1. What is an irrational number?

a. A number that can be expressed as a fraction
b. A number with a finite decimal representation
c. A number with a non-repeating, non-terminating decimal
d. A number that is always positive

2. Which of the following is an example of an irrational number involving a square root?

a. $\frac{1}{2}$ b. $\sqrt{4}$ c. $\sqrt[3]{8}$ d. $\sqrt{7}$

3. Which of the following numbers is an example of an irrational number involving cube roots?

a. 8 b. $\frac{3}{4}$ c. $\sqrt[3]{3}$ d. 2.5

4. Which of the following numbers is an irrational number?

a. $\sqrt{1}$ b. $\sqrt{4}$ c. $\sqrt{9}$ d. $\sqrt{14}$

5. Which number is rational?

a. $\sqrt{11}$ b. 0.949876... c. $\sqrt[3]{27}$ d. π

6. Which of the following is an example of an irrational number that cannot be expressed as the cube root of an integer?

a. $\sqrt[3]{64}$ b. $\sqrt[3]{27}$ c. $\sqrt[3]{10}$ d. $\sqrt[3]{8}$

7. Estimate the value of $\sqrt[3]{12}$ and select the closest approximation from the choices.

a. 1 b. 2 c. 3 d. 4

8. If you estimate the value of $\sqrt{8}$, what is your best estimate?

a. 1 b. 2 c. 3 d. 4

9. If you want to place $\sqrt[3]{10}$ on a number line, which two perfect cubes would you use as reference points for estimation?

a. $\sqrt[3]{8}$ and $\sqrt[3]{9}$ b. $\sqrt[3]{8}$ and $\sqrt[3]{27}$ c. $\sqrt[3]{1}$ and $\sqrt[3]{8}$ d. $\sqrt[3]{4}$ and $\sqrt[3]{16}$

10. If you want to place $\sqrt{15}$ and $\sqrt[3]{20}$ on the same number line, which number would be closer to 2?

a. $\sqrt{15}$
b. $\sqrt[3]{20}$
c. Both are equidistant from 2
d. It cannot be determined without more information



Explore

A store named Irrational Cake Store offers number cakes with a twist. It only accepts orders which are numbers that cannot be expressed as a fraction. One example is a cake that looks like $\sqrt{2}$.

What do you think is so special about the number in the number cake?

You might have noticed that the number contains a similar symbol which we learned during the previous topic – the radical symbol. But unlike the last lesson, 2 is not a perfect square as it cannot be expressed as a product of two integers.

Moreover, $\sqrt{2}$ is a number that cannot be written as a fraction of two integers. When expressed as a decimal number, we can also observe that $\sqrt{2}$ is a number which has non-repeating and non-terminating decimals. These types of numbers are called irrational numbers.

To dive in deeper, here are the key characteristics of irrational numbers:

1. Non-repeating decimals: When you express an irrational number as a decimal, the decimal places go on forever without repeating a pattern. For example, the decimal representation of $\sqrt{2}$ is approximately 1.414213562373095... and it goes on indefinitely without repeating.
2. Non-terminating decimals: Irrational numbers do not terminate, meaning they do not end in a finite number of decimal places. In the same example, the decimal representation of $\sqrt{2}$ which is 1.414213562373095... goes on with no end.
3. Cannot be expressed as fractions: Unlike rational numbers, which can be expressed as fractions, irrational numbers cannot be written as the ratio of two integers. Rational numbers, such as $\frac{1}{2}$ or $\frac{3}{4}$, have finite or repeating decimal representations. We can't express $\sqrt{2}$ as a fraction.

Now that we know what irrational numbers are, let us learn how to approximate their values.

In order to estimate the square root of a number that is not a perfect square, we must identify the two nearest integers to that number. To do this, it is important for us to be familiar with various perfect square values.

Let us try $\sqrt{52}$:

From our previous lesson, we know that 49 is a perfect square, and 7 is its square root. After 49, 64 is the next perfect square, and its square root is 8. Therefore, we can say that $\sqrt{52}$ is between 7 and 8.

To approximate the value of $\sqrt{52}$ further:

Let us get the square of 7.5, which is halfway between 7 and 8.

$$(7.5)^2 = 56.25$$

We almost got the value of 52. We need a smaller number. Let us now compute for the squares of numbers between 7 and 7.5.

$$(7.1)^2 = 50.41$$

$$(7.2)^2 = 51.84$$

$$(7.3)^2 = 53.29$$

$$(7.4)^2 = 54.76$$

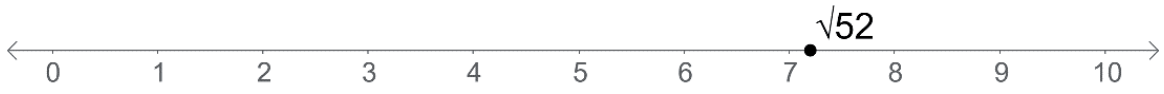
Since 52 is closest to 51.84, then we can say that $\sqrt{52}$ is approximately 7.2.

Now let us try plotting this in a number line.

$\sqrt{52}$ is between 7 and 8, the square roots of 49 and 64.

Since 52 is closer to 49 than to 64, we may just plot $\sqrt{52}$ close to 7.

We also know from our computation earlier that the value of $\sqrt{52}$ is approximately 7.2, so we may place $\sqrt{52}$ around that location in the number line.



Let us try estimating the value of another irrational number, $\sqrt{17}$. The perfect square nearest 17 is 16, and we know that its square root is 4. After 16, the next perfect square is 25, and its square root is 5. Therefore, we can say that $\sqrt{17}$ is in between 4 and 5.

To approximate its value further, we get the square of 4.5, which is halfway between 4 and 5.

$$(4.5)^2 = 20.25$$

We almost got the value of 17. We need a smaller number.

Computing for the squares of numbers between 4 and 4.5, we get the following:

$$(4.1)^2 = 16.81$$

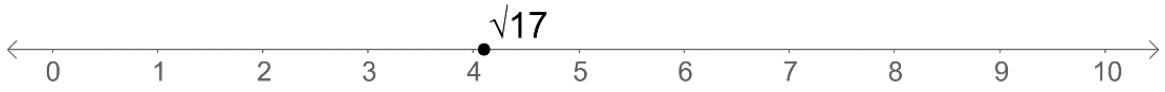
$$(4.2)^2 = 17.64$$

$$(4.3)^2 = 18.49$$

$$(4.4)^2 = 19.36$$

Since 17 is closest to 16.81, then we can conclude that $\sqrt{17}$ is approximately 4.1.

Plotting this in a number line, we get the following:



To estimate the value of an irrational number involving cube roots, we can just use the same technique we did for square roots. Take for example $\sqrt[3]{3}$.

The perfect cubes nearest 3 are 1 and 8. We also know that their cube roots are 1 and 2, respectively.

Therefore, we can say that $\sqrt[3]{3}$ is in between 1 and 2.

To illustrate this, we can write it as follows:

$$\sqrt[3]{1} < \sqrt[3]{3} < \sqrt[3]{8}$$

Now, we estimate the position of $\sqrt[3]{3}$ based on how near it is from 1 or from 8. We get the halfway between 1 and 8 which is 4.5. From this, we can observe that 3 is less than 4.5 So, the estimate we can get is that $\sqrt[3]{3}$ is less than 1.5, which is halfway between 1 and 2.

Plotting this in a number line also just uses the same concept as plotting estimates for irrational numbers involving square roots.

$\sqrt[3]{3}$ is between 1 and 2, the cube roots of 1 and 8. Since 3 is closer to 1 than to 8, we may just plot $\sqrt[3]{3}$ close to 1. Furthermore, since we know that $\sqrt[3]{3}$ is less than 1.5, we can plot it somewhere at that point.





Try This

Now try plotting irrational numbers involving cube roots by yourself. On a blank sheet of paper, draw a number line. Next, mark the approximate position of the cube root of 9 ($\sqrt[3]{9}$) on the number line. Ensure that you apply the same method we discussed earlier for estimating irrational numbers. Illustrate and show your solution clearly.

1. Between which two cube roots is the cube root of 9?
2. Of these two cube roots, which one is closer to the cube root of 9?



Key Points

- Irrational numbers are real numbers that cannot be expressed as a simple fraction or ratio of two integers (whole numbers).
- They have non-repeating, non-terminating decimal expansions. In other words, irrational numbers are numbers whose decimal representations go on forever without repeating a pattern.
- We can estimate the values of irrational square roots and cube roots.



Do

I. Encircle the letter of the correct answer.

1. What is the primary characteristic that distinguishes irrational numbers from rational numbers?

a. They have finite decimal representations
b. They can be expressed as fractions
c. They have repeating decimal patterns
d. They have non-repeating, non-terminating decimals

2. What do you call a number that cannot be expressed as a simple fraction?

a. Rational number
b. Whole number
c. Irrational number
d. Integer

3. Which of the following statements is true regarding irrational numbers?

a. All irrational numbers are whole numbers
b. Irrational numbers can never be expressed as fractions
c. The square root of 16 is irrational
d. The cube root of 8 is an irrational number

4. Which number is an irrational number?

a. 0.5
b. 1.25
c. $\sqrt{5}$
d. $\frac{2}{3}$

5. Which of the following numbers is not an irrational number?

a. π
b. $\sqrt{8}$
c. $\frac{1}{4}$
d. $\sqrt[3]{25}$

6. Which of the following is an example of an irrational number that cannot be expressed as the square root of an integer?

a. $\sqrt{25}$
b. $\sqrt{49}$
c. $\sqrt{66}$
d. $\sqrt{100}$

7. If you estimate the value of $\sqrt[3]{6}$, what is your best estimate?

a. 1
b. 2
c. 3
d. 4

8. Estimate the square root of 13 and select the closest approximation from the choices.

a. 2
b. 3
c. 4
d. 5

9. If you want to place $\sqrt{10}$ and $\sqrt[3]{10}$ on the same number line, which number would be closer to 2?

a. $\sqrt{10}$
b. $\sqrt[3]{10}$
c. Both are equidistant from 2
d. It cannot be determined without more information

10. If you want to place $\sqrt{10}$ on a number line, which two perfect squares would you use as reference points for estimation?

a. 2 and 5 b. 3 and 6 c. 1 and 4 d. 4 and 9

II. Arrange the following irrational and rational numbers in ascending order on a number line.

1. $-5, \sqrt{13}, \frac{-1}{5}, \frac{\sqrt{4}}{2}, \pi$

2. $4, \sqrt{26}, \frac{-7}{4}, \frac{1}{\sqrt{5}}, 3\pi$

III. List down the square roots of numbers from 1 to 20. Identify which ones are rational and which ones are irrational.



Brainstorm

Find a partner within your class. Your teacher will help you. Applying the knowledge you've gained, attempt to approximate the value of the square root of 29. Follow the same procedures that we have previously discussed. Write your solution and answer in a sheet of bond paper. Ensure that your worksheet includes a thorough explanation and a description of your thought process.



Design

Your teacher will group you into teams of 4 members each. Create a poster or infographic illustrating examples of irrational numbers in daily life. They can include images or illustrations demonstrating where irrational numbers are found, explanations of how they are used, and the importance of understanding them in mathematics and the real world.

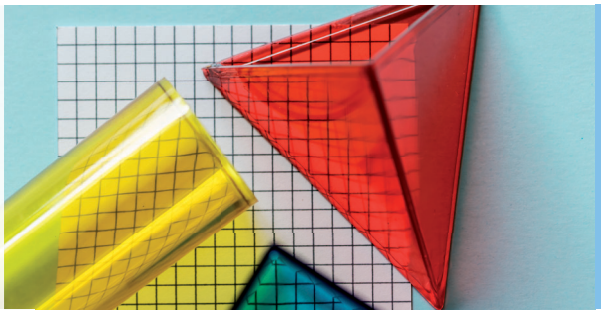


Fun Facts

There's an infinite amount of irrational numbers between any two rational numbers on the number line. Even though we can't list all irrational numbers, they are numerous and densely distributed across the number line.

LESSON 3

Units of Measure



Objectives

- Convert units of measure within the International System of Units (SI).
- Convert between different systems of measure.

Examine

There are so many practical applications of measurement. When following recipes, measurements such as cups, tablespoons, and milliliters are used for ingredients like flour, sugar, water, or oil. Understanding these units is crucial for accurate cooking and baking. What would happen to our food if we just put random amounts of ingredients?

When planning a journey or traveling, distances are measured in kilometers or miles. Understanding these units helps in estimating travel time and fuel consumption.

In interior design or construction, room dimensions and area measurements (square meters or square feet) are crucial. It helps determine the amount of paint required, carpeting, or floor tiles needed for a space.

At the market or grocery store, various items are sold by weight or volume. Understanding units like kilograms, grams, liters, and milliliters is essential for purchasing items like fruits, vegetables, liquids, and more. Does your family have a grocery list? If yes, do your parents put the measurements of the items they need?

Measurement

- Units of Measure
- Volume of Cylinders
- Volume of Pyramids

Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).

In this lesson, we will explore the units of measurement within the different measurement systems. We will also learn how to convert measurements within the International System of Units (SI), as well as other measurement systems.



Recall

Encircle the letter of the correct answer.

1. What is the base unit for measuring length in the International System of Units (SI)?

a. Centimeter b. Meter c. Kilogram d. Liter

2. Which SI prefix represents 10^{-3}?

a. Kilo b. Deci c. Milli d. Centi

3. In the English Metric System, how many inches are in a foot?

a. 10 b. 11 c. 12 d. 13

4. Which of the following is an example of converting from SI units to English units?

a. Kilograms to grams
b. Meters to kilometers
c. Inches to centimeters
d. Gallons to liters

5. When converting from SI units to English units, what is the appropriate unit to measure length?

a. Meters b. Kilometers c. Feet d. Gallons

6. When converting from SI units to English units for mass measurements, which of the following is a common conversion factor?

a. 1 kilogram = 2.2 pounds
b. 1 meter = 39.37 inches
c. 1 liter = 1000 milliliters
d. 1 kilometer = 1000 meters

7. If a recipe calls for 500 g of flour, what is the equivalent amount in kilograms?

a. 0.05 kg b. 0.5 kg c. 5 kg d. 50 kg

8. A swimming pool is 12 m long. Convert this length to centimeters.

a. 120 cm b. 1200 cm c. 12 000 cm d. 120 000 cm

9. The distance between two cities is 480 km. Approximate this distance to miles.

a. 48 mi b. 300 mi c. 480 mi d. 800 mi

10. You have a recipe that requires 250 mL of milk. Convert this volume to ounces.

a. 8.45 oz b. 25 oz c. 0.25 oz d. 0.025 oz



Explore

In almost all our daily activities, we are surrounded with phenomena and substances that can be quantified for. A quantity that has magnitude and units is referred to as a physical quantity.

The process of associating physical quantity and phenomenon with numbers or an appropriate standard or unit is called measurement. This is fundamental in all fields of health and sciences, engineering and construction, and even in our day to day lives. For that reason, the history of the development of measurement, its elements, conditions, and limitations have been studied for a long period of time.

The development of measurement can be traced back from the primitive system used by the people of Indus Valley during the period of 3000-1500 BC. In their system, length is measured as a distance from the forearm from the elbow to the tip of the middle finger. Such length is referred to as cubit. Approximately, one cubit is 45.72 cm.

The earliest units of mass are grains and stones. The stone standard is usually used as larger units to measure body mass. Approximately, one stone is 6.35 kg.

The number of days in a year and time of the day defined by hours, minutes, and seconds are attributed to the Babylonians. It is believed that the 360 days in a year was likely similar to a circle divided into 360 degrees.

These developments in measurement brought about the establishment of the International Bureau of Weights and Measures (BIPM), an organization that maintains the most commonly used metric system called the **International System of Units** (SI units). Some of the common SI units are meter (m) for length, kilogram (kg) for mass, and second (s) for time.

For very large or very small quantities, SI prefixes are used to indicate multiplication of the physical quantities. The table below shows the summary of commonly used prefixes.

Prefix	Symbol	Multiple
giga	G	10^9
mega	M	10^6
kilo	k	10^3
centi	c	10^{-2}
milli	m	10^{-3}
micro	u	10^{-6}

Now that we already have an idea of what measurement is, let us explore the units of measurement used for length.

The basic unit for length is meter (m) and the other units for length are kilometer (km), centimeter (cm), and millimeter (mm). The table shows the conversion of the other units in relation to meters.

Other units of length	Value in meters
1 km	1000 m
1 cm	0.01 m
1 mm	0.001 m

Take for example 2.5 km, how would you convert this length to meters?

Remember that 1 km = 1000 m.
So, we can say that:

$$2.5 \text{ km} = 2.5(1000 \text{ m})$$
$$2.5 \text{ km} = 2500 \text{ m}$$

How about converting 2400 cm to meters?

Similar to what we did earlier, remember that 1 cm = 0.01 m.
Therefore, we can say that:

$$2400 \text{ cm} = 2400(0.01 \text{ m})$$
$$2400 \text{ cm} = 24 \text{ m}$$

Now, let us try expressing 1.23 m to millimeters.

Again, remember that 1 m = 1000 mm.
So, we can say that:

$$1.23 \text{ m} = 1.23(1000 \text{ mm})$$
$$1.23 \text{ m} = 1230 \text{ mm}$$

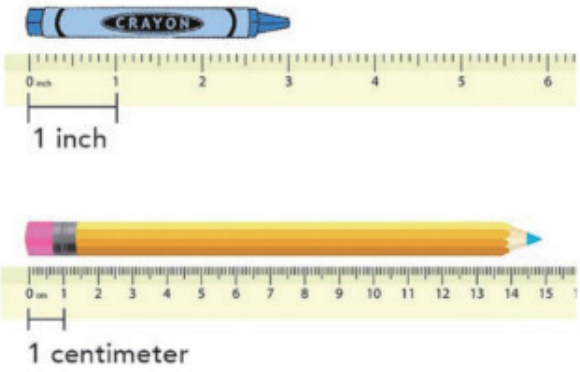
Lastly, let us try converting 5.1 km to millimeters.

Remember that 1 km = 1000 m, and 1 m = 1000 mm.
Therefore:

$$5.1 \text{ km} = 5.1(1000)(1000 \text{ mm})$$
$$5.1 \text{ km} = 5\,100\,000 \text{ mm}$$

Other than the SI units, we also have what we call the English Metric System.

One example to illustrate this is by observing a ruler. The ruler is commonly used to measure short lengths. When measuring using the ruler, the one end of the object is placed (tip of the ruler). The eye is placed directly above the ruler.



Looking back at the ruler, its one side is expressed in centimeters, while the other side in inches. This is just one example of a unit in the English Metric System. This is the system which other countries use in order to measure and quantify things. Other units used in this system are foot, yard, or mile. Here is a list of the conversions of the English Metric System and SI units.

English Metric System	Conversion Metric	SI Units
1 inch (in)		2.54 cm
1 foot (ft)	12 in	30.50 cm
1 yard (yd)	3 ft	0.915 m
1 mile (mi)	5280 ft	1.609 km

Let us try expressing 21 yd to feet.

Remember that 1 yd = 3 ft.

$$21 \text{ yd} = 21(3 \text{ ft})$$
$$21 \text{ yd} = 63 \text{ ft}$$

How about converting 2.75 mi to meters?

Remember that 1 mi = 1.609 km and 1 km = 1000 m.

$$2.75 \text{ mi} = 2.75(1609 \text{ m})$$
$$2.75 \text{ mi} = 4424.75 \text{ m}$$

Another basic quantity is called mass. It is the amount of matter in an object which is usually measured using mass balances or scales.

In SI units, mass is measured in kilograms (kg), and its basic unit is gram (g). Meanwhile, the unit of mass in the English metric system is pound (lb).

SI Units
1 t (tonne or metric ton or Megagram) = 1000 kg
1 kg = 1000 g
1 mg (milligram) = 0.001 g
English Metric System
1 lb = 16 oz (ounces) = 454 g
1 ton = 2200 lb

Let us try converting 0.0002 t to grams.

Remember that 1 t = 1000 kg, and 1 t = 1 000 000 g.
So, we can say that:

$$0.0002 \text{ t} = 0.0002(1\,000\,000 \text{ g})$$
$$0.0002 \text{ t} = 200 \text{ g}$$

How about expressing 19 lb to grams?

Remember that 1 lb = 454 g.
Therefore:

$$19 \text{ lb} = 19(454 \text{ g})$$
$$19 \text{ lb} = 8626 \text{ g}$$



Try This

Individually, you will have to create a brochure on the different conversion rates from the English Metric System to SI Units System for the following aspects: length and mass. As you are creating a brochure, make sure to express your creativity in this activity.

1. What are the conversion rates for length?
2. What are the conversion rates for mass?



Key Points

- Physical quantity is a quantity that has magnitude and units.
- Measurement is the process of associating physical quantity and phenomenon with numbers or an appropriate standard or unit.
- The International System of Units, or the SI units, is the most commonly used metric system. This is maintained by an organization called the International Bureau of Weights and Measures (BIPM). Some of the common SI units are meter (m) for length, kilogram (kg) for mass, and second (s) for time.
- The English Metric System is another metric system which is used by other countries. Some common English units are inch (in) and foot (ft) for length, as well as pound (lb) and ounce (oz) for mass.



Do

Encircle the letter of the correct answer.

1. In the SI system, what is the basic unit for measuring mass?

a. Gram b. Kilogram c. Liter d. Meter

2. In the English Metric System, how many feet are in a yard?

a. 2 b. 3 c. 4 d. 5

3. What is the metric prefix for 1000?

a. Kilo b. Milli c. Centi d. Deci

4. What is the relationship between the International System of Units and the English Metric System?

a. They are identical systems.
b. They have no connection; they are entirely separate.
c. SI units is based on English units.
d. They are related but have different base units.

5. Which of the following is not an example of converting from SI units to English units?

a. Kilograms to grams
b. Gallons to liters
c. Kilograms to pounds
d. Meters to yards

6. When converting from SI units to English units for length measurements, which of the following is a common conversion factor?

   a. 1 meter = 1 foot
   b. 1 meter = 10 centimeters
   c. 1 inch = 2.54 centimeters
   d. 1 kilogram = 1 pound

7. If you have a length of 20 cm, what is its equivalent in millimeters?

   a. 2 mm b. 200 mm c. 2000 mm d. 20 000 mm

8. You have a mass of 150 g. Convert this mass to kilograms.

   a. 0.15 kg b. 1.5 kg c. 15 kg d. 150 kg

9. If a textbook weighs 2.5 kg, convert its weight to pounds.

   a. 5.5 lb b. 2.25 lb c. 22.05 lb d. 0.25 lb

10. A swimming pool is 25 m long. Approximate this length to yards.

    a. 22.5 yd b. 27.34 yd c. 28.5 yd d. 50 yd



Brainstorm

First, you need to invent and come up with your own conversion problem. Afterwards, partner up with a classmate. Challenge your partner to perform and solve the conversion problem that you did. Lastly, verify the correctness of his/her answer.



Design

Your teacher will group you into teams of 4 members each. The SI units use 20 prefixes, 6 of which were already introduced to you earlier. In this group research challenge, you will need to research with your groupmates to complete the rest of the prefixes used by the International System of Units.

Prefix	Symbol	Multiple	English Word of the Decimal Form
yotta	Y	10^{24}	septillion
zetta		10^{21}	sextillion
		10^{18}	

peta			quadrillion
tera	T		
giga	G	10^9	billion
mega	M	10^6	million
kilo	k	10^3	thousand
	h		hundred
deca	da	10	ten
deci	d		
centi	c	10^{-2}	hundredth
milli	m	10^{-3}	thousandth
micro	u	10^{-6}	millionth
nano			billionth
pico		10^{-12}	
femto	f		
atto	a		quintillionth
zepto		10^{-21}	
yocto	y	10^{-24}	

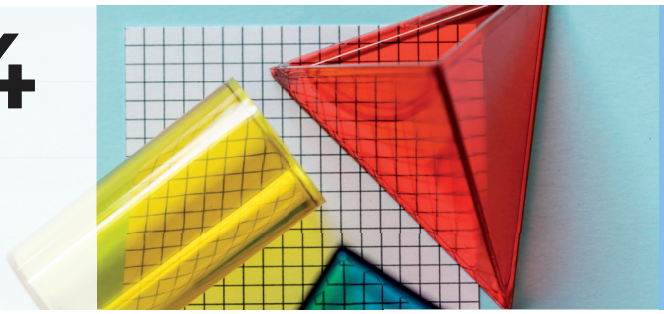


Fun Facts

The metric system was first introduced during the French Revolution. It aimed to create a standardized system based on natural units. The meter, for example, was initially intended to be one ten-millionth of the distance from the equator to the North Pole along a meridian passing through Paris.

LESSON 4

Volume of Cylinders



Objectives

- Explain inductively the volume of a cylinder using the area of a circle, leading to the identification of the formula.
- Find the volume of a cylinder.
- Solve problems involving the volumes of cylinders.

Examine

In this lesson, our focus will be on cylinders. We will delve into understanding how to determine the volume of a cylinder using its formula. We will also practice computing cylinder volumes, as well as putting this knowledge into action to solve practical, real-world challenges.

Measurement

Units of Measure

Volume of Cylinders

Volume of Pyramids

Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).



Recall

Encircle the letter of the correct answer.

1. What is the formula for finding the area of a circle?

a. $A = 2\pi r$ b. $A = \pi r^2$ c. $A = 2r^2$ d. $A = \pi r$

2. How do you calculate the volume of a cylinder?

a. $V = \pi r$ b. $V = \pi r^2 h$ c. $V = \pi r^3$ d. $V = 2\pi rh$

3. In finding the volume of a cylinder, what does π represent?

a. Perimeter b. Diameter c. Radius d. Pi

4. What is the primary concept behind finding the volume of a cylinder?

a. Calculating the area of the curved surface
b. Measuring the height only
c. Determining the distance around the circular base
d. Understanding how much space it occupies

5. What does the height of a cylinder represent in the volume formula?

a. The length of the cylinder's sides
b. The distance from one circular base to the other
c. The distance across the circular base
d. The distance around the cylinder

6. If the radius of a cylinder is doubled while keeping the height the same, how does this affect the volume?

a. The volume doubles
b. The volume is halved
c. The volume quadruples
d. The volume remains the same

7. If the radius of a cylinder is 5 cm and the height is 10 cm, what is its volume?

a. 650 cm^3 b. 180 cm^3 c. 250 cm^3 d. 785 cm^3

8. What is the volume of a cylinder with a height of 3 cm and a radius of 3 cm?

a. 66.73 cm^3 b. 75.03 cm^3 c. 84.78 cm^3 d. 96.76 cm^3

9. If the volume of a cylinder is 1000 cm^3 and its height is 10 cm, what is its approximate radius?

a. 6 cm b. 7 cm c. 8 cm d. 9 cm

10. You have a cylindrical storage tank with a radius of 8 m. If you want to store 1000 m^3 of liquid, what should be the approximate height of the tank?

a. 3 m b. 4 m c. 5 m d. 6 m

Explore

What can you observe with the pictures? Can you give some similarities among them?



You guessed it, right! These objects have similar shapes. This shape is called a cylinder. It has two flat circular faces (bases) and a curved surface that connects these bases.

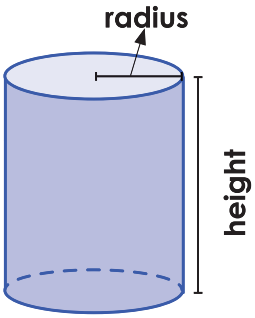
Before we dive deeper into the world of cylinders, let us have a quick recap on circles. This will be helpful for us as we have noted that a cylinder has circles for its bases.

Do you still remember how to compute for the area of a circle? We use the formula $A = \pi r^2$, where A is the area, and r is the radius. We use the value of 3.14 for π.

With this knowledge, we can now move on to discover the volume of cylinders.

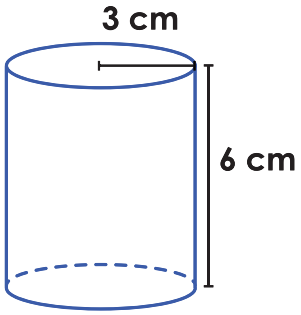
Similar to other prisms, the volume of a cylinder is calculated by multiplying the area of its base by its height.

$V = \pi r^2 h$



Since we know that the base of a cylinder is a circle, we can say that the area of the base of the cylinder is πr^2. We then multiply this to the height (h) of the cylinder gives us its volume.

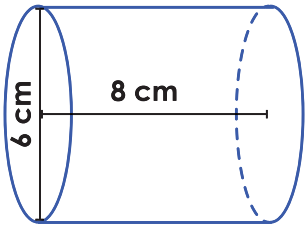
Let us get the volume of the cylinder that has a 3 cm radius and a 6 cm height.



$V = \pi r^2 h$
$V = 3.14(3^2)(6)$
$V = 169.56 \text{ cm}^3$

We conclude that the volume of the cylinder is 169.56 cm^3.

Let us try another one. Let us solve for the volume of the cylinder below.



From the figure, the diameter of 6 cm is given and we can solve that the radius is 3 cm. The height is 8 cm.

Solving for the volume,

$V = \pi r^2 h$
$V = 3.14(3^2)(8)$
$V = 226.08\ cm^3$

Therefore, we can say that the volume of the cylinder is 226.08 cm^3.

Let us now explore where we can apply the concept of the volume of cylinders in real life.

Suppose that a family decided to start saving water by collecting rainwater in a drum. The family bought 3 cylindrical drums with a base of 1 m (diameter) and a height of 1.5 m. What is the total volume of rainwater that the family can collect?

First, we identify the given in the problem.

3 cylindrical drums
Base (diameter) = 1 m
Height = 1.5 m

Next, we state what is asked in the problem.

The total volume of rainwater that the family can collect in their drums

Now, we solve for the asked.
We identify the radius of the cylinder first.

r = 0.5 m

Next, we solve for the volume of one drum.

$V = \pi r^2 h$
$V_{1\ drum} = 3.14(0.52)(1.5)$
$V_{1\ drum} = 1.1775\ cm^3$

Now that we know that the volume of a drum is 1.1775 cm^3, we can get the total volume of the 3 drums.

$V_{3\ drums} = 3(1.1775)$
$V_{3\ drums} = 3.5325\ cm^3$

Lastly, we write our complete answer to the problem.

The total volume of rainwater that the family can collect is 3.5325 cm^3.



Try This

Following the steps that we discussed on how to solve problems involving the volume of a cylinder, answer the problem. Make sure to show what is given, the asked, your solution, and your complete answer. A drum has a volume of 76.36 ft^3. Approximately, how many pails of water can fill the drum if the pail is 2 ft in diameter and 1 ft in height?

1. What are given?
2. What is asked?
3. What is the final answer?



Key Points

- Finding the volume of a cylinder is similar to finding the volume of other prisms. You just need to multiply the base area by its height.
- The formula for the volume of a cylinder is $V = \pi r^2 h$, where V is the volume, r is the radius, and h is the height of the cylinder.



Do

I. Encircle the letter of the correct answer.

1. What shape do the two circular bases of a cylinder form?

a. Square b. Rectangle c. Circle d. Triangle

2. In finding the volume of a cylinder, what does h represent?

a. Perimeter b. Diameter c. Height d. Radius

3. What information is needed to calculate the volume of a cylinder?

a. Height and perimeter
b. Height and circumference
c. Radius and height
d. Diameter and circumference

4. How is the volume of a cylinder different from its surface area?

a. They are the same thing
b. Surface area involves the height, while volume does not
c. Volume is three-dimensional, while surface area is two-dimensional
d. Surface area is calculated using π, while volume is not

5. What does understanding the volume of cylinders allow you to do in practical scenarios?

a. It has no real-life applications
b. Calculate how much liquid a container can hold
c. Estimate distances
d. Determine the perimeter of a circle

6. How does the volume of a cylinder change if the height is doubled while keeping the radius the same?

a. The volume is halved
b. The volume remains the same
c. The volume becomes four times larger
d. The volume doubles

7. What is the volume of a cylinder with a radius of 3 cm and a height of 8 cm?

a. 226.08 cm^3 b. 106.19 cm^3 c. 255 cm^3 d. 348 cm^3

8. What is the volume of the cylinder with a radius of 5 cm and a height of 8 cm?

a. 628 cm^3 b. 689 cm^3 c. 736 cm^3 d. 778 cm^3

9. If you have a cylinder with a volume of 250 cm^3 and a height of 5 cm, what is its approximate radius?

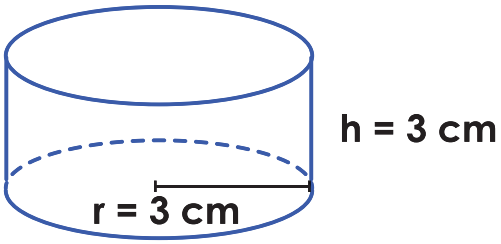
a. 3 cm b. 4 cm c. 5 cm d. 6 cm

10. You have a cylindrical water container with a radius of 3 in and a height of 12 in. Approximately how many gallons of water can it hold? (hint: 1 gallon = 231 cubic inches)

a. 1.47 gal b. 2.47 gal c. 1.86 gal d. 2.86 gal

II. Solve for the volume of the following cylinders.

1.



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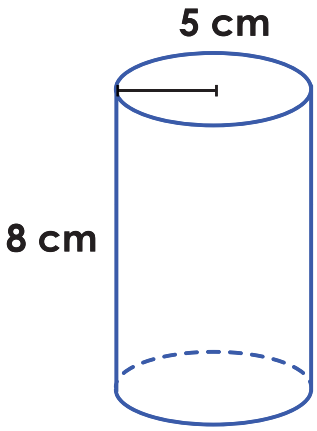
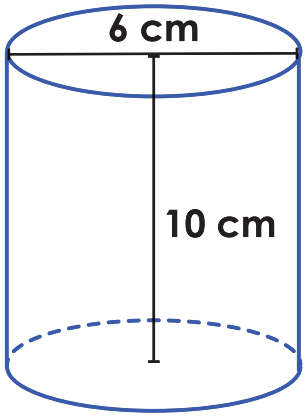
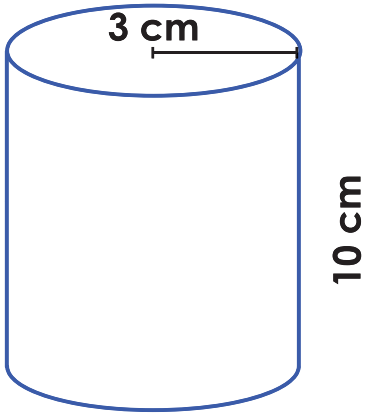


3.



4.



III. Solve the following problems. Draw a sketch of the cylinder.

1. A family decided to start saving water by collecting rain water in a drum. The family bought 3 drums with a base of 0.5 m and a height of 1.5 m. What is the total volume of rain water that the family can collect?
2. A drum has a volume of 76.36 cubic feet. How many pails of water can fill the drum if the pail is 1 foot in diameter and 1 foot in height?
3. What is the volume of a test tube with a radius of 1 cm and a height of 8 cm?



Brainstorm

First, find a partner within your class. Your teacher will help you. Make sure that you have your ruler with you since you will be using it for this activity.

As a pair, your mission is to find an object that has a cylindrical shape. Measure both its radius and height. Afterwards, calculate for its volume using the formula that we have previously discussed. Write your solution and answer on a piece of bond paper.



Design

Your teacher will group you into teams of 4 members each. Bring cardboard or paper and create your own cylinders. You can cut out circles and rectangles, assemble them to form cylinders, and then measure and calculate the volumes.

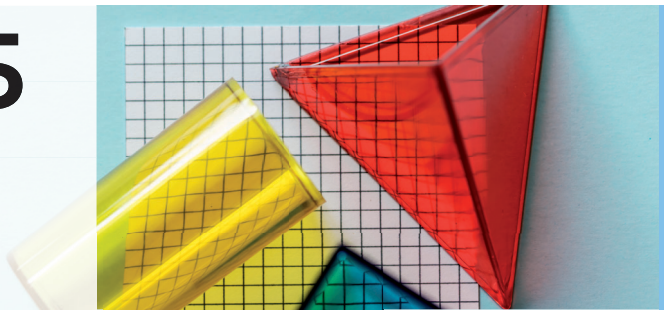


Fun Facts

Cylindrical containers, such as cans and bottles, are designed to minimize the materials used while maximizing their storage volume. Companies use mathematical principles to optimize these containers' shapes for efficiency.

LESSON 5

Volume of Pyramids



Objectives

- Explore inductively the volume of pyramids using prisms, leading to the identification of the formula.
- Estimate volumes of pyramids.
- Solve problems involving volumes of pyramids.

Examine

Pyramids are seen all around us. Back then, Egyptians built pyramids. These have historical significance and are architectural marvels. Some roofs today are pyramid-shaped. Would you want your future house to have a pyramid-shaped roof?

There are pyramid-shaped food items like cheese wedges or cake pieces. What kind of food do you want to see be shaped like a pyramid?

We can also relate the concept of a food pyramid (nutrition guide) to a geometric pyramid. The layers of the food pyramid and how its volume, akin to a pyramid in geometry, represents different proportions of food groups in a healthy diet.

Measurement

Units of Measure

Volume of Cylinders

Volume of Pyramids

Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).

In this lesson, we will be focusing on another type of 3d figure – pyramids. We will learn and understand how to determine the volume of a pyramid using its formula. Furthermore, we will

engage in exercises to calculate pyramid volumes and apply this newfound understanding to tackle real-world problems.



Recall

Encircle the letter of the correct answer.

1. What is the formula for finding the area of a square?

a. $A = s^2$ b. $A = s^3$ c. $A = lw$ d. $A = \frac{1}{2}bh$

2. What is the formula for finding the area of a triangle?

a. $A = s^2$ b. $A = s^3$ c. $A = lw$ d. $A = \frac{1}{2}bh$

3. What is the formula for calculating the volume of a pyramid?

a. $V = lwh$ b. $V = Bh$ c. $V = \frac{1}{3}Bh$ d. $V = \pi r^2 h$

4. When getting the volume of a pyramid, which of the following is a key factor you need to consider?

a. The height of the pyramid
b. The number of triangular faces
c. The length of the pyramid
d. The color of the pyramid

5. What happens to the volume of a pyramid if you double its height while keeping the base area the same?

a. It is halved
b. It remains the same
c. It is quadrupled
d. It doubles

6. How does the volume of a pyramid change if you increase the length of its base sides while keeping the height constant?

a. The volume increases
b. The volume decreases
c. The volume remains the same
d. The volume cannot be determined with this information

7. You have a triangular pyramid with a base area of 25 in^2 and a height of 10 in. What is its volume?

a. 55.67 in^3 b. 52 in^3 c. 81 in^3 d. 83.33 in^3

8. What is the volume of the rectangular pyramid below with a base length of 8 cm, base width of 7 cm, and height of 6 cm?

a. 112 cm^3 b. 222 cm^3 c. 312 cm^3 d. 333 cm^3

9. A rectangular pyramid has a volume of 160 cm^3. If the height of the pyramid is 4 cm, what is the area of its base?

a. 110 cm^2 b. 120 cm^2 c. 130 cm^2 d. 140 cm^2

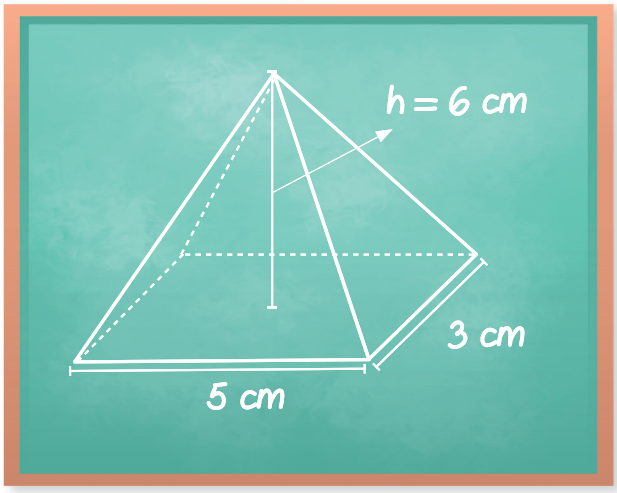
10. A square pyramid has a volume of 500 cm^3. If the length of one side of its base is 10 cm, what is the height of the pyramid?

a. 5 cm b. 10 cm c. 15 cm d. 20 cm



Explore

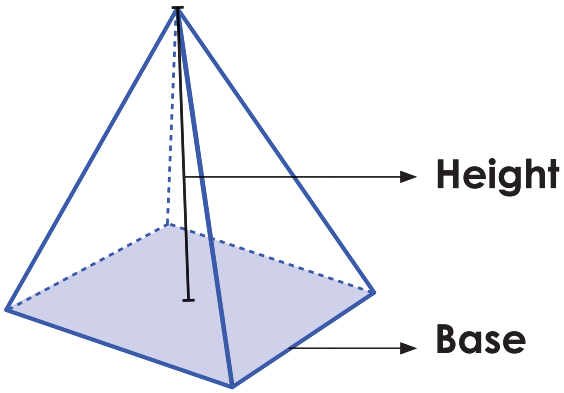
Ryan and his sister saw a container of juice in their dining table. It was a pyramid-shaped container that they weren't used to seeing. Can you help them find out how much juice it contains if it has the following dimensions:



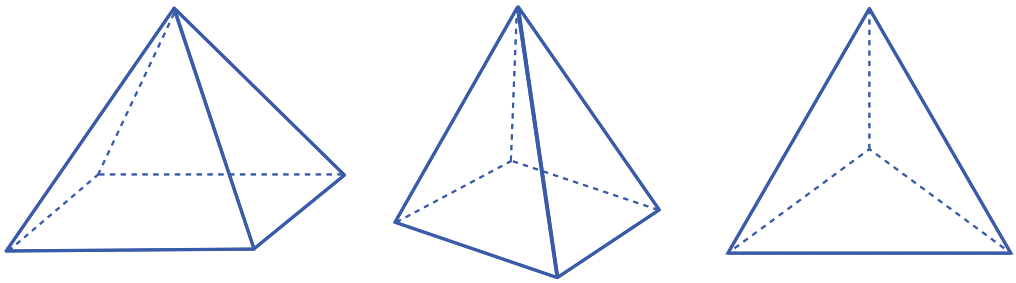
The volume of a pyramid is given by the following formula:

$$V = \frac{1}{3}Bh$$

where B is the area of its base and h is its height



Remember that a pyramid can have a rectangular, square, or triangular base.



Take note that pyramids and prisms may sometimes look similar, especially when they have the same base and height. However, their volumes are different. The volume of a pyramid is only one-third of the volume of a prism when they share the same base and height. Understanding this relationship helps us appreciate the differences and similarities between various geometric shapes.

Going back to the container of juice that Ryan and his sister saw, the base of the pyramid has dimensions of 5 cm by 3 cm and the height of the pyramid is 6 cm. This is a rectangular pyramid. We can now compute for its volume.

First, find the area of the base (B) of the pyramid. Multiply the area of the base (B) by the height (h) of the pyramid. Then, divide the product by 3.

$$V = \frac{1}{3}Bh$$

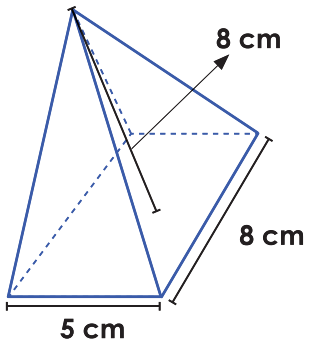
$$V = \frac{1}{3}[5(3)](6)$$

$$V = 30$$

Therefore, the volume of the juice container is 30 cm^3.

Take note that the areas of the bases differ based on the shape. For a square base, we multiply side by side. For a rectangular base, we multiply length by width. For a triangular base, we get half of the base times height.

Let us try another example. Let us solve for the volume of the pyramid:



Take note that this pyramid is a rectangular pyramid as the base is a rectangle.

$$V = \frac{1}{3}Bh$$

$$V = \frac{1}{3}[5(8)](8)$$

$$V = 106.67$$

Therefore, the volume of the pyramid is 106.67 cm^3.

Let us now explore where we can apply the concept of the volume of pyramids in real life.

If the base of the Great Pyramid of Giza has an area of 55 000 m^2, and its height is 146 m, can you solve for the volume of the pyramid?

First, we identify the given in the problem.

Base = 55 000 m^2
Height = 146 m

Next, we state what is asked in the problem.

The volume of the Great Pyramid of Giza

Now, we solve for the asked.
We solve for the volume of the pyramid.

$$V = \frac{1}{3}Bh$$

$$V = \frac{1}{3}(55000)(146)$$

$$V = 2\ 676\ 666.67$$

Lastly, we write our complete answer to the problem.

The volume of the Great Pyramid of Giza is 2 676 666.67 m^3.

We can also estimate the volume of pyramids. Using the same problem, let us round to the highest place values and solve.

$$V = \frac{1}{3}(60000)(146)$$

$$V = 2\ 000\ 000$$

The estimated volume of the Great Pyramid of Giza is 2 000 000 m^3. This is close to the actual value.



Try This

Following the steps that we discussed on how to solve problems involving the volume of a pyramid, answer the problem. Make sure to show what is given, the asked, your solution, and your complete answer.

Imagine you are a nutritionist working to create a food pyramid to represent a balanced diet for a healthy lifestyle. Your task is to calculate the volume of a food pyramid-shaped exhibit at a museum to ensure that it can accommodate a certain amount of food items.

You are designing a food pyramid exhibit with a square base measuring 5 m on each side and a height of 4 m. Calculate the volume of the food pyramid to determine how much space it provides for food items.

1. What are given?
2. What is asked?
3. What is the final answer?



Key Points

- Finding the volume of a cylinder is similar to finding the volume of other prisms. You just need to multiply the base area by its height. The volume of the pyramid is $\frac{1}{3}$ of the volume of its enclosing prism.
- The formula for the volume of a pyramid is $V = \frac{1}{3}Bh$, where V is the volume, B is the base area, and h is the height.



Do

I. Encircle the letter of the correct answer.

1. What is the formula for finding the area of a rectangle?

   a. $A = s^2$ b. $A = s^3$ c. $A = lw$ d. $A = \frac{1}{2}bh$

2. What information is needed to calculate the volume of a pyramid?

   a. Height and perimeter
   b. Height and area of base
   c. Radius and height
   d. Diameter and circumference

3. In finding the volume of a pyramid, what does h represent?

a. Perimeter b. Diameter c. Height d. Radius

4. Which of the following factors primarily affects the volume of a pyramid?

a. The length of its edges
b. The area of its base and its height
c. The number of vertices it has
d. The color of its surface

5. If you have two pyramids with different base shapes but the same height and volume, what must be true about these pyramids?

a. They have the same base area
b. They have the same number of edges
c. They have the same slant height
d. They have the same base side lengths

6. What happens to the volume of a pyramid if you double both its base area and its height?

a. The volume quadruples
b. The volume doubles
c. The volume remains the same
d. The volume is halved

7. If the base of a pyramid has a length of 6 m and a width of 4 m, and the height is 8 m, what is its volume?

a. 96 m^3 b. 64 m^3 c. 192 m^3 d. 32 m^3

8. What is the volume of the pyramid with a base length of 8 cm, a base width of 5 cm, and a height of 8 cm?

a. 106.67 cm^3 b. 153.33 cm^3 c. 100 cm^3 d. 150 cm^3

9. You have a triangular pyramid with a volume of 90 in^3. If the height of the pyramid is 6 in, what is the area of its triangular base?

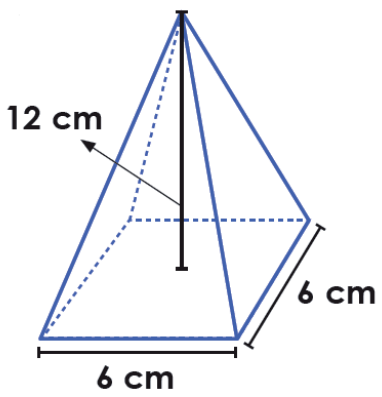
a. 30 in^3 b. 35 in^3 c. 40 in^3 d. 45 in^3

10. You have a square pyramid with a volume of 1200 m^3. If the area of its base is 300 m^2, what is the height of the pyramid?

a. 10 m b. 11 m c. 12 m d. 13 m

II. Find the volume of the following pyramids.

1.



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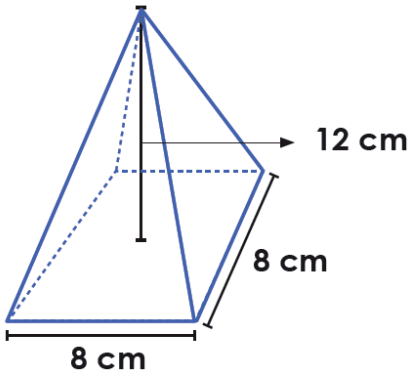

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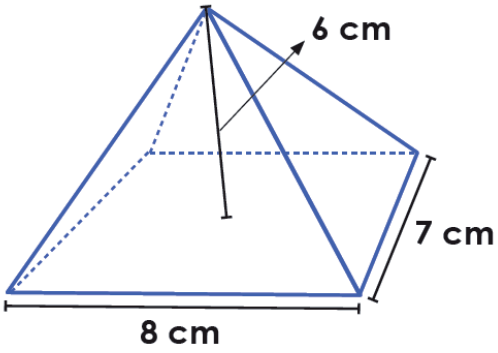
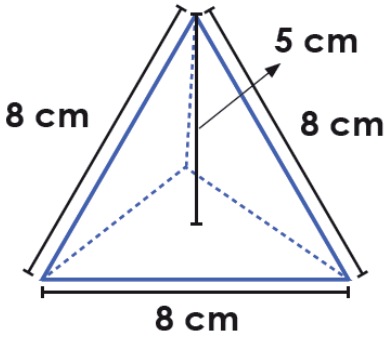


4.



III. Solve the following word problems. Sketch the pyramid.

1. A pyramid has a square base with 4-cm sides and a height of 9 cm. Find the volume of the pyramid.
2. Find the volume of a rectangular pyramid having a base with dimensions 8 cm by 6 cm and a height of 5 cm.
3. How much milk is contained in a square-pyramid shaped container that has a base with 6-cm sides and a height of 8 cm?
4. The base of the Great Pyramid of Giza has an area of 55 000 square meters. Its height is 146 m. What is the pyramid's volume?
5. A rectangular pyramid has a base with dimensions 20 cm by 40 cm and a height of 20 cm. What is its volume?



Brainstorm

Your teacher will pair you up with a partner. Bring materials like paper, cardboard, or clay to create various pyramid shapes. Once constructed, measure the base and height of each pyramid and calculate their volumes.



Design

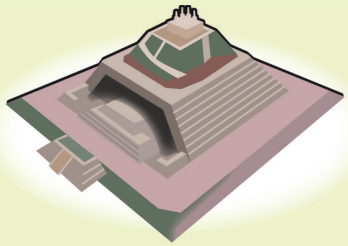
Form a team of three. Your teacher will help you. For each student in your group, the task is to create a pyramid illustration. Draw your artwork on a sheet of bond paper.

Make sure that each of you draws one triangular pyramid, one square pyramid, and one rectangular pyramid. Do your best to give your drawings a three-dimensional appearance. You also need to identify and assign a value to the pyramid's base and height. Afterwards, share and present your creation with your fellow groupmates. Get the volumes of the pyramids of your groupmates.



Fun Facts

The largest pyramid in the world is the Great Pyramid of Cholula in Puebla, Mexico. It has a volume of 4.45 million cu. m.



LESSON 6

Sets



Objectives

- Describe sets and their subsets, the union of sets, and the intersection of sets.
- Illustrate sets and their subsets, the union of sets, and the intersection of sets, using Venn diagrams.

Examine

Sets can be seen in the various types of smart phones. Have you ever compared the features of your smart phone with your friends? Each brand or model forms a distinct set, and there could be an intersection set representing smart phones that possess features from multiple brands.

In this lesson, we will delve into the fundamental concept of sets and explore how sets can be related to each other through subsets, unions, and intersections. We will also learn to represent these relationships visually using Venn diagrams.

Sets and Real Numbers

Sets

Real Numbers

Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).



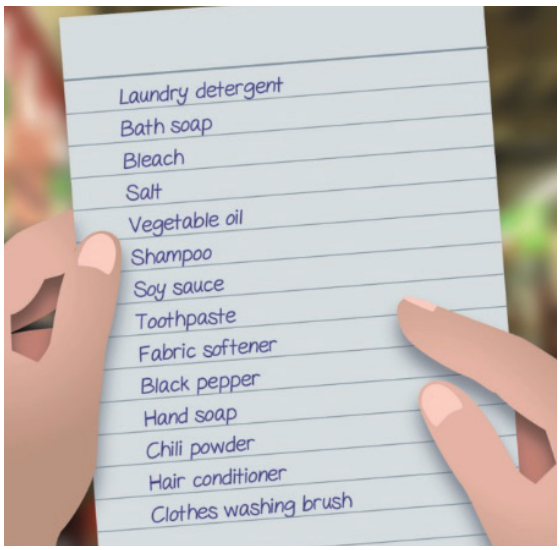
Recall

When your family goes to the grocery, do you have a shopping list? Can you categorize the types of items you will buy?



Explore

Your mother needs to buy some stuff for the household. To prevent her from getting lost in the store, she asks for your help to rearrange the items on her grocery list in such a way that it is organized into different categories. How are you going to rearrange the following items on the list?



Our notion of sets is not different from how it is defined in mathematics. A set is a well-defined collection of distinct objects or things called elements. Well-defined means that the elements of a set share a common property which helps us to determine if an object belongs to that set or not.

In our given problem, each category you create can be an example of a set:

- a set of personal care products, which includes bath soap, shampoo, toothpaste, hand soap, and hair conditioner
- a set of basic cooking ingredients, which includes salt, vegetable oil, soy sauce, black pepper, and chili powder
- a set of laundry products, which includes laundry detergent, bleach, fabric softener, and clothes washing brush

We can identify the items or products on the list which are in one category and those which belong to another.

For us to have a deeper understanding of sets, let us learn how we can best represent them.

Sets are usually named using capital letters such as A, B, or C, while the elements of a set are usually denoted by small letters such as a, b, or c.

The symbol "$\in$" is used to denote that an object is "an element of the set" while "$\notin$" is used to say that an object is "not an element of the set".

For example, let P be the set of personal care products written in the grocery list in our given problem. Then we can write: bath soap $\in$ P and chili powder $\notin$ P.

The set P can be represented in many different ways. One way is by enclosing a complete list of its elements in a pair of braces "{}", that is: P = {bath soap, shampoo, toothpaste, hand soap, hair conditioner}. This is called the list or roster method. Here, the elements of the set are written in any order and separated by commas. The elements also must be listed only once.

Another way of representing the set P is given by: P = {x | x is a personal care product in the grocery list}. Here, x stands for any element of P. The symbol "|" is read as "such that", and the phrase describes the property or rule that each element of the set must satisfy. We read the notation of set P as "the set of all x such that x is a personal care product on the grocery list". This representation is called the rule method or set-builder notation.

Here are some more examples for you to explore:

1. A set B of basic cooking ingredients in the grocery list can be represented in the roster form, written as B = {salt, vegetable oil, soy sauce, black pepper, chili powder}.
2. The set L = {laundry detergent, bleach, fabric softener, clothes washing brush} can be written in set-builder notation as L = {x| x is a laundry product in the grocery list}.
3. Consider the set X consisting the numbers 2, 4, 6, 8, and 10. Its roster form can be written as X = {2, 4, 6, 8,10}, while its set-builder notation can be expressed as X = {x | x is an even number from 2 to 10}.

Now that we know how to represent sets properly, let us discuss what equal sets are, and what a subset means.

Consider the following sets below:

A = {x | x is a vowel in the English alphabet}
B = {x | x is a letter in the English alphabet}
C = {a, e, i, o, u}

Compare the elements of the sets A, B, and C. What can you say on the relationships between the given sets?

In the given sets above, A and C are equal since the elements of C are precisely the vowels of the English alphabet. In symbols, we write A = C.

To define this further, let X and Y be sets. We can say that X and Y are equal, written X = Y, if both sets have the exact same elements.

Moreover, if you noticed that every element of A also belongs to B, then you are right. Because of this, we can say that A is a subset of B, or $A \subseteq B$.

Defining this, if X and Y are sets, then X is a subset of Y, written $X \subseteq Y$, if each element of X is also an element of Y.

Going back to our grocery list from earlier, if we take the set of all items in the grocery list as our universal set U, then, we can say that sets P (personal care products), B (basic cooking ingredients), and L (laundry products) are subsets of U.

Let us dive deeper into the world of sets. Did you know that sets also have operations? Some of these operations are the intersection of sets and the union of sets.

The intersection of two sets is the set of elements that are common to both sets. Thus, if X is {1, 2} and Y is {2, 3, 4}, then the intersection of sets X and Y is: $X \cap Y = \{2\}$. Symbolically, the intersection of X and Y is denoted by $X \cap Y$.

Meanwhile, the union of two sets is the set of elements that belong to one or to both of the two sets. Thus, from the same sets X and Y, the union of sets X or Y is: $X \cup Y = \{1, 2, 3, 4\}$. Symbolically, the union of X or Y is denoted by $X \cup Y$.

Let us explore what we have learned so far by practicing with an example.

Suppose that at home, you have a hobby of collecting books since your dream is to have your mini library. How are you going to classify the following titles of books to sort them? What books are common to mathematics and languages?

Beauty and the Beast	Watching Your Diet
The Encyclopedia	Essay Writing in a Convenient Way
Speak English Right	Cooking the Right Way
The New Testament	How to Have Your Body Fit
You and Your Environment	Table of Random Numbers
The Old Testament	The Circulatory System
Basic Writing: Your Letters and Numbers	Genetics

We note that finding books that can be found in two or more classifications show the intersection of the sets while having the whole mini library with all the books together is the union of the sets.

A set of language books are: Beauty and the Beast, Speak English Right, Basic Writing: Your Letters and Numbers, The Encyclopedia, and Essay Writing in a Convenient Way

A set of mathematics books are: The Encyclopedia, Basic Writing: Your Letters and Numbers, and Table of Random Numbers

A set of science books are: You and Your Environment, Cooking the Right Way, How to Have Your Body Fit, The Circulatory System, Watching Your Diet, The Encyclopedia, and Genetics

A set of health books are: You and Your Environment, Cooking the Right Way, The Circulatory System, The Encyclopedia, Watching Your Diet, and How to Have Your Body Fit

A set of spirituality books are: The Encyclopedia, The New Testament, and The Old Testament

From this, let us first represent our sets.

A = Language books

A = {The Encyclopedia, Beauty and the Beast, Speak English Right, Basic Writing: Your Letters and Numbers, and Essay Writing in a Convenient Way}

B = Mathematics books

B = {The Encyclopedia, Basic Writing: Your Letters and Numbers, and Table of Random Numbers}

C = Science books

C = {You and Your Environment, Cooking the Right Way, How to Have your Body Fit, The Circulatory System, Watching Your Diet, The Encyclopedia, and Genetics}

D = Health books

D = {You and Your Environment, How to Have Your Body Fit, Cooking the Right Way, The Circulatory System, Watching Your Diet, and The Encyclopedia}

E = Spirituality books

E = {The Encyclopedia, The New Testament, and The Old Testament}

Now, let us find the intersection of language books and mathematics books.

We know that set A = language books and set B = mathematics books

The intersection of language books and mathematics books can be written as A ∩ B.

From what we have learned, we know that to find the intersection of the sets, we need to identify the common elements between them. The common elements between sets A and B are The Encyclopedia, and Basic Writing: Your Letters and Numbers. Expressing this in proper representation, we can say that $A \cap B = \{x \mid x \text{ is a language and a mathematics book}\}$. Furthermore, $A \cap B = \{\text{The Encyclopedia, and Basic Writing: Your Letters and Numbers}\}$.

How about finding the union of all the books in the mini library? This includes all language, science, health, spirituality, and mathematics books.

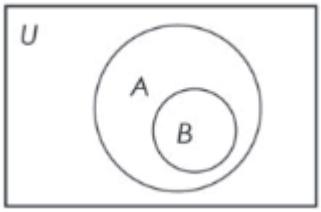
We know that set A = language books, set B = mathematics books, set C = science books, set D = health books, and set E = spirituality books.

The union of all the books in the mini library can be written as $A \cup B \cup C \cup D \cup E$.

From what we have learned, we know that to find the union of the sets, we need to identify the elements that belong to sets A or B or C or D or E. Expressing this in proper representation, we can say that $A \cup B \cup C \cup D \cup E = \{$Beauty and the Beast, The Encyclopedia, Speak English Right, The New Testament, You and Your Environment, The Old Testament, Basic Writing: Your Letters and Numbers, Watching Your Diet, Essay Writing in a Convenient Way, Cooking the Right Way, How to Have Your Body Fit, Table of Random Numbers, The Circulatory System, Genetics$\}$.

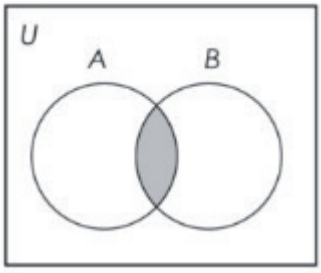
Let us now discuss how to represent sets using Venn diagrams. The Venn diagram consists of a rectangle that represents the universal set and circles inside the rectangle which represent the set or sets being considered in the discussion. It is understood that the elements in the set are inside the circle that represents the sets.

In the figures, we let A and B be sets, while U is the universal set.



$B \subseteq A$

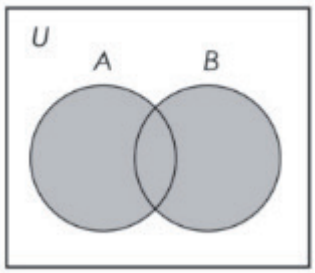
This figure shows that set B is a subset of set A, denoted by $B \subseteq A$. As we have previously discussed, B is a subset of A as all the elements in B also belongs to A.



$A \cap B$

This shows the intersection of the sets A and B, denoted by A ∩ B. This consists of the shared regions of A and B.

The figure shows the union of the sets A and B, denoted by A ∪ B. This is the set that contains those elements that belong to A, B, or to both.



A ∪ B



Try This

Based on our mini library exercise earlier, draw a Venn diagram that shows sets C and D. Make sure that you are able to identify the relationship between the two sets, as well as their intersection and union.

1. What are the elements that are common to both sets C and D?
2. What are all the elements that are found in set C, set D, and both sets C and D?



Key Points

- A set is a well-defined collection of objects called elements.
- Sets can be represented using list or roster method and rule method or set-builder notation.
- Sets A and B are equal if both sets have the same elements.
- A set A is a subset of a set B if each element of A is also an element of B.
- The intersection of two sets is the set of elements that are common to both sets.
- The union of two sets is the set of elements that belong to one or both of the two sets.
- Venn diagrams help us visualize sets.



Do

Encircle the letter of the correct answer.

1. Which of the following represents the concept of a subset?

a. $A \cup B$ b. $A \cap B$ c. $A \subseteq B$ d. $A = B$

2. In a Venn diagram, where are the elements that belong to both sets placed?

a. Inside the first circle
b. Inside the second circle
c. In the overlapping area
d. Outside both circles

3. What is the symbol for the intersection of two sets?

a. $\cap$ b. $\subseteq$ c. $\cup$ d. $\in$

4. Which operation results in a set that contains all the elements from both sets but removes duplicates?

a. Intersection b. Union c. Subset d. Complement

5. What does it mean when we say set A is a subset of set B?

a. Set A contains all elements of set B
b. Set A has some elements in common with set B
c. All elements of set A are also in set B
d. Set A and set B have no elements in common

6. Which operation would you use to find all the unique elements in the combined sets A and B?

a. Intersection b. Complement c. Union d. Subset

7. If set $X = \{2, 4, 6\}$ and set $Y = \{3, 6, 9\}$, what is $X \cup Y$?

a. $\{2, 3, 4, 6, 9\}$ b. $\{2, 4, 6\}$ c. $\{3, 9\}$ d. $\{6\}$

8. If set $M = \{1, 2, 3, 4, 5\}$ and set $N = \{3, 4, 5, 6, 7\}$, what is $M \cap N$?

a. $\{1, 2\}$ b. $\{3, 4, 5\}$ c. $\{1, 2, 6, 7\}$ d. $\{3, 4, 5, 6, 7\}$

9. If set S = {red, blue, green} and set T = {blue, yellow, purple}, what is the result of $S \cap T$?

a. {red}
b. {blue}
c. {green, yellow, purple}
d. { }

10. If set M = {Monday, Wednesday, Friday} and set N = {Wednesday, Saturday, Sunday}, what is the result of $M \cup N$?

a. {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}
b. {Wednesday}
c. {Monday, Wednesday, Friday, Saturday, Sunday}
d. {Saturday, Sunday}



Brainstorm

Pair up with a classmate. Your teacher will help you. Collaboratively create Venn diagrams using digital tools or apps using Sets D and E from the mini library exercise from the Try This.



Design

Form groups consisting of four members per team. Your teacher will help you. Decide on a topic that everyone in your group is interested in. It could be about movies, songs, animals, or any other topic you all like.

Each group member should make their own sets based on the chosen topic. Write down your sets neatly, and ensure that you use the proper representation and notations.

Each member will take a turn to show their sets to the group. Explain why you chose the things in your sets and how they relate to the topic.



Fun Facts

The symbols ∪ and ∩ are like cups.

∪ would hold more water than ∩.

Hence, the union is the one with more elements than the intersection.

LESSON 7

Real Numbers



Objectives

- Illustrate the different subsets of real numbers.

Examine

We have discussed before about the uses of rational and irrational numbers in real life. Real numbers contain both rational and irrational numbers. Here are some situations. Temperature readings, distance, speed, money, expenses, weight, measure, population, and data analysis are just some examples that involve real numbers. Which of these fields is most interesting for you?

In this lesson, we will delve into the fascinating world of real numbers and their subsets. We will be engaged through interactive activities, such as creating visual representations of these subsets. And we will also gain a deeper understanding of the practical applications of real numbers in everyday life.

Sets and Real Numbers

Sets

Real Numbers

Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).



Recall

Encircle the letter of the correct answer.

1. What is a rational number?

a. A number with a non-repeating, non-terminating decimal
b. A number that can be expressed as a fraction
c. A number with an imaginary part
d. A whole number

2. Which type of number has a decimal representation that never ends and never repeats?

a. Rational number
b. Irrational number
c. Whole number
d. Natural number

3. Which subset of real numbers includes numbers like π?

a. Whole numbers
b. Rational numbers
c. Irrational numbers
d. Natural numbers

4. Which of the following is an example of a rational number?

a. $\sqrt{3}$ b. -0.75 c. π d. $\sqrt{2}$

5. Which of the following numbers is NOT a real number?

a. $\frac{1}{2}$ b. -3 c. 0.75 d. $\sqrt{-1}$

6. Which subset of real numbers includes numbers like 2, -5, and 0?

a. Rational numbers
b. Natural numbers
c. Irrational numbers
d. Whole numbers

7. What is the decimal representation of the rational number $\frac{3}{4}$?

a. 0.75 b. 1.5 c. 0.25 d. 0.333...

8. You have a pizza that is divided into 8 equal slices. If you eat 3 slices, what fraction of the pizza have you consumed?

a. $\frac{1}{8}$ b. $\frac{2}{8}$ c. $\frac{3}{8}$ d. $\frac{4}{8}$

9. Which of the following is a rational number with a negative value?

a. $\sqrt{8}$ b. $\sqrt{25}$ c. -1.5 d. π

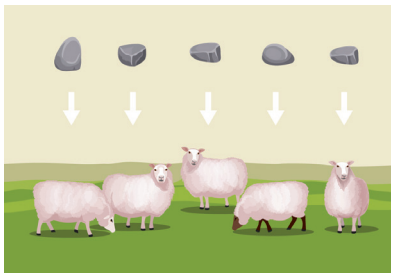
10. If you add a rational number and an irrational number, what type of number do you get?

a. Always rational
b. Always irrational
c. Sometimes rational, sometimes irrational
d. It's impossible to determine



Explore

Describe the two pictures. What do you think is the common among the two pictures?

	
Counting sheep using stones	Ancient Egyptians dividing pieces of bread among their laborers

In the early days, people had already the concepts of numbers though they did not have yet the symbols that we have today. They used the concept of numbers to count their possessions like animals using stones. They also used it to distribute something equally among themselves like food, wages, etc. This has been very useful and meaningful especially to our daily lives. Numbers also have evolved over time. These are now written in universal symbols and are classified into categories.

Natural numbers are the numbers that we use for counting objects.

1, 2, 3, 4, 5, ...

Note that the three dots at the end (ellipsis) indicate that the list keeps going forever.

We use natural numbers to describe or count things around us:

- 12 male teachers
- 25 computers
- 60 armchairs
- 50 students
- 10 female teachers
- 2156 heart reactions

Let's say a blogger has posted a video on his website. A week passed, there were no views at all. Then, we can say that the blogger's video has zero number of views that week. The idea of "zero" or nothing came to be considered as a number. Whole numbers are the set of natural numbers plus the number zero.

0, 1, 2, 3, 4, 5, ...

We use whole numbers to describe or count things around us:

- no assignment today (0 assignments)
- no class today due to typhoon
- 5 friends
- 300 brave men
- 1000 days

How could we have less than zero? Is there anything like negative number of views of a video? None. However, this concept of negative is shown when you owe something. For example, you get zero balance in your pocket then you borrow PhP 10 from your classmate. How much do you have? You have negative PhP 10. So, this abstract idea has been accepted and now called negative numbers. Then, this set of numbers including whole numbers and negative version of counting numbers is called the integers. Integers will be further discussed in the next quarter.

..., -4, -3, -2, -1, 0, 1, 2, 3, 4, ...

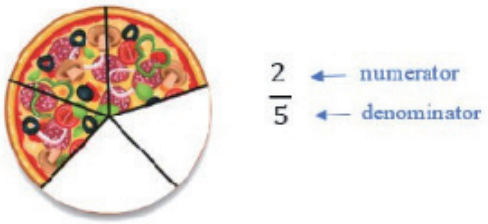
We use integers to describe the things around us:

- A balance of two hundred fifty pesos from a cash loan (it means -PhP 250)

The next idea that we are going to meet is the idea of the fraction. Though this does not apply to the number of people, number of objects like chairs, computers; many other things in real life are measured in fractions, like a half-meter of cloth and one-fourth gallon of milk.

If we add fractions to the set of integers, we get the set of rational numbers. We already discussed this the previous quarter but here is a review. These consist of any numbers of the form $\frac{a}{b}$, where a and b are integers (but b cannot be zero). Rational numbers include what we call fractions. Since the word "rational" comes from the word "ratio", which pertains to fractions.

Fractions have the top and bottom parts. The bottom part is called the denominator which tells how many parts the whole is divided equally. The top part is called the numerator which tells how many parts of the whole are taken.



Remember the denominator cannot be zero, but the numerator can. Any fraction whose numerator is zero, the whole fraction is equal to zero. It means no part of the whole has been taken. However, if the denominator is zero, it does not make sense since the fraction has no parts at all.

All integers can also be thought of as rational numbers, with a denominator of 1. $5 = \frac{5}{1}$.

Hence, the previous sets of numbers (natural numbers, whole numbers, and integers) are subsets of the rational numbers.

Fractions can be represented by decimals: terminating and repeating decimals.

$$\frac{1}{2} = 0.5 \qquad \frac{1}{3} = 0.333\ldots$$

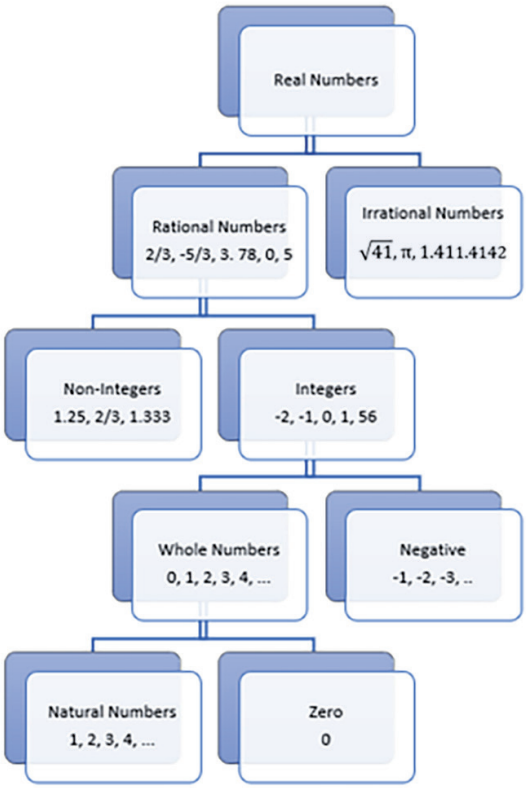
We use rational numbers to describe things in real life:

- 60% download complete is $\frac{60}{100}$.
- 3 out of 22 pages printed out is $\frac{3}{22}$.

As we have previously discussed in the previous lessons, there are numbers that cannot be expressed as fractions, and these are called irrational numbers. As decimals, they never repeat or terminate.

π = 3.1415926536... (the decimals never repeat or terminate)

When we put together rational numbers and irrational numbers, then we will have the complete set of real numbers. This includes any number or points on a number line. The diagram shows the relationships of the set that make up the real numbers.

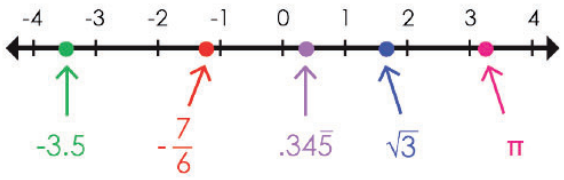


Any two different numbers on a number line can be compared as one is greater or lesser than the other.

For any two real numbers a and b, one and only one of the following three statements is true:

- a is less than b, (expressed as a < b)
- a is equal to b, (expressed as a = b)
- a is greater than b, (expressed as a > b)

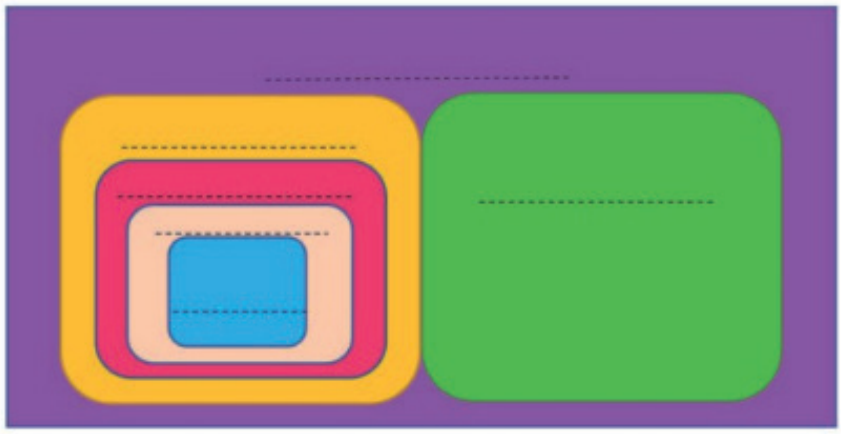
Each point on a number line has a corresponding real number. All numbers to the right of zero are positive real numbers. While to the left of zero are negative numbers.





Try This

Show the relationships among natural or counting numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers using the Venn diagram below. Fill each broken line with its corresponding answer.



1. What is the relationship between natural numbers and whole numbers?
2. What is the relationship between whole numbers and integers?
3. What is the relationship between integers and rational numbers?



Key Points

- Natural numbers/counting numbers are the numbers we use in counting things, that is {1, 2, 3, 4, ...}.
- Whole numbers are numbers consisting of the set of natural numbers and zero.
- Integers are a set of numbers including whole numbers and the negative versions of counting numbers.
- Rational numbers consist of any numbers of the form $\frac{a}{b}$, where a and b are integers (but b cannot be zero). This set includes integers plus fractions.
- Irrational numbers are numbers that cannot be expressed as fractions. As decimals, they never repeat or terminate.
- Real numbers consist of the complete set of rational and irrational numbers. This includes any number or points on a number line. This includes any number or points on a number line.



Do

Encircle the letter of the correct answer.

1. What is the set of all rational and irrational numbers called?

a. Imaginary numbers
b. Complex numbers
c. Rational numbers
d. Real numbers

2. On a number line, where would you find the real numbers?

a. Between -1 and 1
b. All along the number line
c. Only to the right of zero
d. To the left of zero

3. Which subset of real numbers includes integers, fractions, and decimals that terminate or repeat?

a. Natural numbers
b. Whole numbers
c. Rational numbers
d. Irrational numbers

4. What makes an irrational number different from a rational number?

a. Irrational numbers have repeating decimals
b. Rational numbers have non-repeating decimals
c. Irrational numbers cannot be expressed as fractions
d. Rational numbers cannot be found on a number line

5. Which of the following is an example of an irrational number?

a. $\sqrt{5}$
b. $\frac{2}{3}$
c. 0.25
d. -4

6. Which of the following numbers is neither rational nor irrational?

a. -3
b. 0
c. $\sqrt{-1}$
d. $\frac{2}{3}$

7. Calculate the value of $\sqrt{9}$.

a. 0
b. 1
c. 2
d. 3

8. You have a cake that is divided into 10 equal slices. If you eat 4 slices, what fraction of the cake have you consumed?

a. $\frac{1}{2}$
b. $\frac{1}{10}$
c. $\frac{4}{10}$
d. $\frac{5}{8}$

9. Which of the following numbers is both rational and a whole number?

a. -1.5
b. 0
c. $\sqrt{2}$
d. $\frac{2}{3}$

10. If you multiply a rational number and an irrational number, what is the result?

a. Always a whole number
b. Always an irrational number
c. Always a natural number
d. It depends on the specific numbers being multiplied



Brainstorm

Find a partner from your class. Your teacher will help you. Take a nature walk (or use images from your teacher if not possible). Identify examples of real numbers in the environment. This could include measuring the heights of trees, observing temperatures, or even counting the number of leaves on a branch.



Design

Form a group consisting of 3 students. Your teacher will help you. Assign a number to each team member (1 to 3). Depending on your designated number, respond to the specified question:

1. Describe 5 things on social media using natural numbers. What things are countable?
2. Describe a thing in your community using the concept of zero or nothing. What is something that is not there?
3. Give a situation which shows the use of integers (zero, positive, or negative numbers). What integer is used to describe the situation?

Ensure you engage in a comprehensive discussion and explanation of your answer with your fellow group members.



Fun Facts

Early number systems all have one thing in common. They require someone to write down many symbols to record a single number and create new symbols for each larger number.

A positional system allows you to reuse the same symbols, by assigning the symbols different values based on their position in the sequence.

Unit Performance Task: Roots Booklet, Volume Experiment, and Sets Presentation

Your teacher will group you into teams of 4 members each. Work together to do the following.

Component 1: Roots and Number Lines

Create a roots and real numbers booklet. List and explain square roots of perfect squares and cube roots of perfect cubes up to a certain number (e.g., 100 for square roots and 50 for cube roots).

Illustrate and explain the concept of 4 irrational numbers using square and cube roots.

Create a number line displaying both rational and irrational numbers (3 each), highlighting the positions of specific square and cube roots.

Component 2: Volume and Measurement Conversion

Conduct a volume exploration experiment.

Use 2 cylindrical and 2 pyramid-shaped objects to experimentally find the volume using water displacement or other practical methods.

Convert the measurements obtained into different units within the SI system and to other systems.

Deduce the formulas for the volume of cylinders and pyramids based on the experiments.

Solve and present a series of problems involving the volumes of these shapes.

Component 3: Sets and Venn Diagrams

Develop a sets and Venn diagrams presentation.

Define sets, subsets, unions, intersections, and real numbers subsets.

Create 4 Venn diagrams illustrating various set relationships. Use topics you learned from other subjects such as *kolonyalismo* and *imperyalismo* from *Araling Panlipunan* and mitosis and meiosis from Science.

Solve and explain 4 problems involving sets using Venn diagrams.

Class Presentation

Each group will present their findings and explanations for each component of the project.

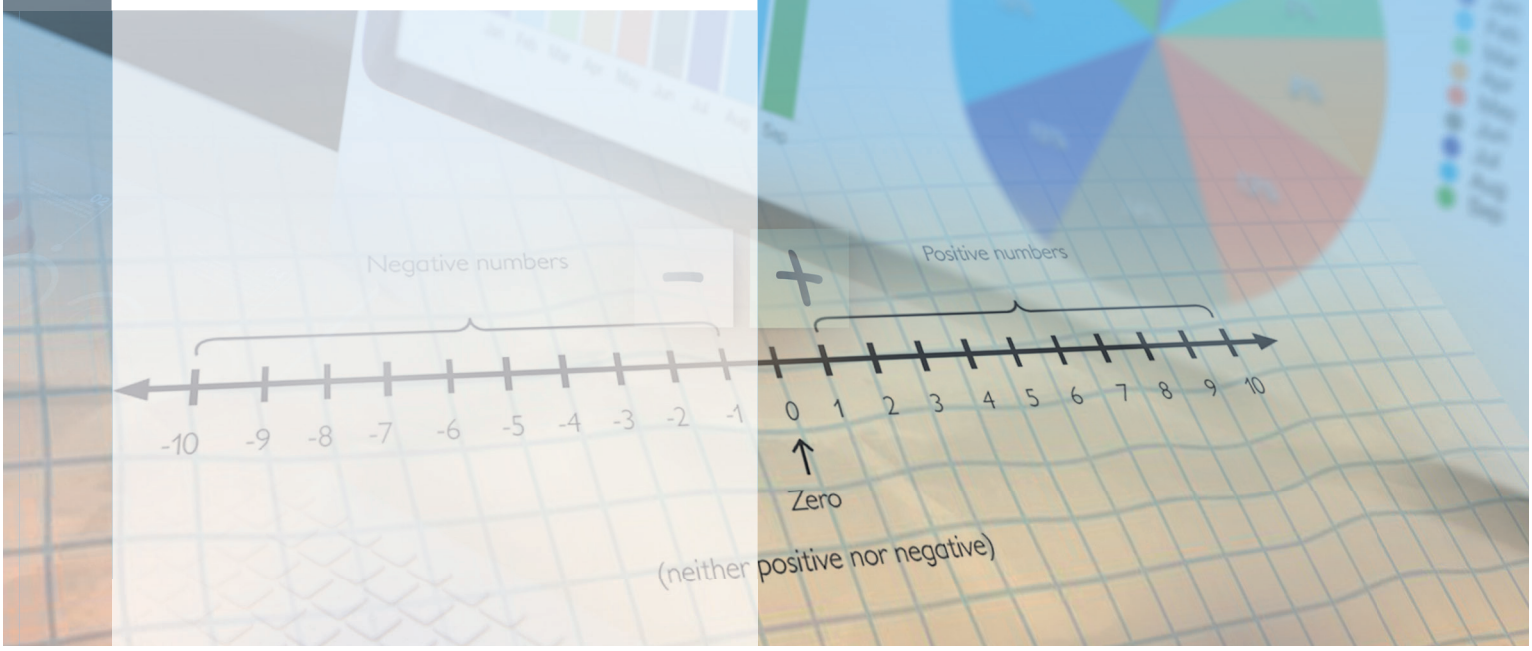
Presentations should include visual aids (booklets, diagrams, models) and be interactive.

Rubric

Criteria	Excellent (4)	Proficient (3)	Developing (2)	Beginning (1)
Roots and Number Line Accuracy	Correct identification and clear explanation of roots; accurate number line representation.	Mostly correct identification; good explanation and number line representation.	Some inaccuracies in identification; basic explanation and number line.	Incorrect identification; poor or no explanation and number line.
Volume Calculation & Conversion	Precise volume calculations; accurate and diverse unit conversions.	Mostly accurate volume calculations; good unit conversions.	Some errors in volume calculations; basic unit conversions.	Incorrect volume calculations; poor or no unit conversions.
Set Theory Understanding	Comprehensive understanding of sets and subsets; clear and accurate Venn diagrams.	Good understanding; mostly clear and accurate Venn diagrams.	Basic understanding; some inaccuracies in Venn diagrams.	Poor understanding; inaccurate or no Venn diagrams.
Presentation & Explanation	Engaging, clear presentation with thorough explanations and confident responses.	Good presentation; clear explanations and good responses.	Adequate presentation; some unclear explanations and hesitant responses.	Poor presentation; unclear explanations and weak responses.

UNIT 3

The exploration continues in the fascinating realm of Mathematics! Throughout this unit, you'll venture further into the domain of data and integers. You'll discover different methods for gathering data and sample collection, organizing data through tables, bar graphs, pie graphs, and other visual aids, as well as deriving valuable insights from these graphs. Moreover, you'll define and arrange integers, conduct various operations on them, and simplify numerical expressions involving these numbers. Finally, you'll delve into understanding the absolute value of integers. By the unit's conclusion, you'll have gained a comprehensive grasp of integers, their operations, and their significance in the mathematical world. Additionally, you'll be well-prepared to collect, represent, and interpret data effectively, a crucial skill applicable across numerous real-life scenarios. Let's proceed, Grade 7 math enthusiasts!



LESSON 1

Collecting Data





Objectives

- Investigate different data collection and sampling techniques.
- Organize statistical data in a frequency distribution table.



Examine

We can collect data on many things. For example, we can find the data of the place of birth of our classmates. How will you collect this data? What are the steps which you should follow for you to accomplish this task? Do you need to have materials for this task?

In this lesson, we will investigate different data collection and sampling techniques. Also, we will organize statistical data in a frequency distribution table.

Data

- Collecting Data
- Graphing Data
- Interpreting Data



Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).



Recall

Encircle the letter of the correct answer.

1. What refers to any information which may be used for a particular purpose?

a. Data
b. Data gathering
c. Sample
d. Sampling

2. Which of the following is a type of non-probability sampling?

   a. Simple random sampling
   b. Systematic sampling
   c. Stratified random sampling
   d. Purposive sampling

3. Which of the following is a method of data collection?

   a. Writing a poem
   b. Solving algebraic equations
   c. Conducting a survey
   d. Painting a landscape

4. Why is it important to choose the appropriate data collection technique for a research study?

   a. To make the research more complex
   b. To ensure the collected data is relevant and accurate
   c. To make the research process longer
   d. To add more colors to the data charts

5. When organizing data into a frequency distribution table, what is the primary objective?

   a. To make the table look colorful
   b. To ensure the table has many rows and columns
   c. To present data in a manner that reveals patterns or trends
   d. To create more work for the researcher

6. Which of the following examples of data can be collected appropriately and conveniently through surveys?

   a. Experience of a crew working at McDonalds
   b. Preferred Milktea flavors of people
   c. Opponents of Gilas Pilipinas in Fiba World Cup 2023
   d. Effects of using a body soap in washing the face

7. Your class wants to find out the favorite subject of students in your grade. Which data collection technique would be the most appropriate?

   a. Interviewing the school principal
   b. Conducting a survey among your classmates
   c. Observing what books students read during breaks
   d. Reviewing the school yearbook

8. You conducted a survey in your school to find out how many pets each student has. You collected the following data: 2, 3, 3, 4, 2, 5, 2, 4, 3. How would you organize this data into a frequency distribution table?

   a. List each number once and count how many times it appears
   b. Add all the numbers and put the total in the table
   c. Write the highest number at the top and the lowest at the bottom
   d. Organize the numbers from least to greatest and then average them

9. Jasmine is examining a frequency distribution table of monthly rainfall in her town. She notices that there are more entries for 20-30 mm of rainfall than any other range. Which of the following conclusions can Jasmine correctly infer?

   a. It always rains 20-30 mm every month
   b. The town receives more rainfall than any other town
   c. The most common amount of rainfall each month is between 20-30 mm
   d. It will rain 20-30 mm next month

10. Which of the following statements is true?

   a. Sample refers to any information which may be used for a particular purpose
   b. Convenience sampling is a type of non-probability sampling
   c. The data about the effects of using body soap in washing the face can be collected appropriately and conveniently through surveys
   d. The data about the effects of increasing the price of rice on the income of sellers can be collected appropriately from participants through probability sampling



Explore

Data refers to any information which may be used for a particular purpose. These data may be gathered in many ways. Some of the data-gathering procedures are survey, interview, registration, and experiment.

A survey is a data-gathering procedure in which the respondents will answer a questionnaire provided by the data collector. There are examples of data that can be collected appropriately and conveniently through surveys. Some of these are food preferences of students, levels of motivation in studying, etc. In terms of gathering data about the food preferences of students, the survey is the right choice. There are many categories under food preferences. Also, there are many students as respondents.

An interview is a data-gathering procedure that involves a presentation or oral-verbal stimuli in terms of oral-verbal responses. There are examples of data that can be collected appropriately and conveniently through interviews. Some of these are the experiences of a person during Martial Law, the secret recipe of a chef in cooking adobo, etc. To gather data about the experiences of a person during Martial Law, asking questions to a person who lived during Martial Law is the right choice. More information will be collected by asking questions from a person.

Registration is one of the data-gathering procedures in which the data is obtained using available references. There are examples of data that can be collected appropriately and conveniently through registrations. Some of these are ways to lose body fat, the life cycle of a butterfly, etc. The data about ways to lose body fat is already available in health books.

An experiment is one of the data-gathering procedures in which the data is obtained through testing using a measuring tool, an intervention material, or a strategy. There are examples of data that can be collected appropriately and conveniently through registrations. Some of these are the weight of a person, the academic performance of students in mathematics, etc. The data about the weight of a person can be obtained by using a weighing scale.

These data-gathering procedures are still inconvenient if the data is to be collected from all respondents in a specific location. To resolve this, sampling should be applied. Sampling refers to the process of selecting individual members from the whole population in which the data is to be gathered. Hence, a sample refers to the subset of individuals from the whole population.

There are two types of sampling which are probability and non-probability sampling. Probability sampling is a type of sampling in which each individual member has an equal chance to be included as one of the respondents. Non-probability sampling is a type of sampling in which each individual member does not have an equal chance to be included as one of the respondents.

Probability sampling has further classifications. Some of these are simple random sampling, systematic sampling, and stratified random sampling.

Simple random sampling is a type of probability sampling in which individual members are selected randomly by means of drawing. Here, the names of participants are drawn randomly on a container. This sampling technique may be applied appropriately in the selection of participants to gather data about the number of drug users in school for example. The selection of participants in drug testing should be done randomly so that there will be no cheating done by the participants.

Systematic sampling is a type of probability sampling in which individual members are selected randomly at regular intervals. Here, all individual members are listed orderly. Then, they are selected based on their order. For example, the data collector selected all number 10 individual members to participate after they were counted repeatedly from 1 to 15.

Stratified random sampling is a type of probability sampling in which individual members are selected randomly based on their groups from the whole population. These groups are classified based on similar characteristics of each individual member of the whole population. The number of samples of one group is always based on the product between its population and the proportion of its population to the total population.

Non-probability sampling also has further classifications. Some of these are purposive sampling and convenience sampling.

Purposive sampling is a type of non-probability sampling in which members are selected based on the purpose of data gathering. This sampling technique may be applied appropriately in the selection of participants to gather data about the effectiveness of eating apples in lowering the level of colon cancer of patients. Here, the participants should be the patients who have colon cancer.

Convenience sampling is a type of non-probability sampling in which members are selected based on the ease of access of the data collector. This sampling technique may be applied appropriately in the selection of participants to gather data about the problems encountered by parents in raising from baby to child. Here, relatives and neighbors can be the participants.

Organizing data into a frequency distribution table helps provide a clear summary of raw data and makes patterns or trends more apparent. Here's a step-by-step guide on how to organize statistical data in a frequency distribution table:

1. Gather Your Raw Data:

   Start by collecting all the data you want to organize. This could be survey results, experimental outcomes, measurements, or other forms of data.

2. Sort the Data:

   Sort the data from the smallest value to the largest value. This makes it easier to count frequencies.

3. Determine the Range:

   Find the range of your data by subtracting the smallest data value from the largest data value.

4. Choose Appropriate Intervals/Bins:

   Decide how many intervals or bins you want in your table. Common choices are between 5 and 15, depending on the spread of the data. Once you've decided the number of bins, divide the range by the number of bins to get the bin width.

   For instance, if your data ranges from 1 to 100 and you choose 10 bins, then each bin might represent a range of 10 units: 1-10, 11-20, and so on.

5. Create the Table:

   Draw a table with at least two columns. The first column will represent the intervals (or bins) and the second column will represent the frequency of data points falling into each bin.

6. Tally the Data:

   Go through your sorted data and place a tally mark in the appropriate interval for each data point. For instance, if you have a data point of 15 using the bins, it would fall into the 11-20 bin.

7. Count the Tallies:

   Once you've tallied all of your data points, count the tally marks for each interval and write down the total in the 'frequency' column.

8. Review and Interpret:

   Once you've completed your table, review it to understand the distribution of your data. Look for patterns, such as intervals with particularly high or low frequencies.

And that's it! With a frequency distribution table, you can quickly see how your data is distributed and identify any patterns or trends in the data.



Try This

Identify which sampling technique (probability or non-probability) is more appropriate to use in selecting respondents to obtain data on the effectiveness of a mathematics remedial summer camp in improving the mathematical performance of students.

1. What is the goal of getting the data?
2. Which of the 2 sampling techniques would help us achieve the goal?



Key Points

- Surveys, interviews, registrations, and experiments are part of the data gathering procedure.
- There are many sampling techniques. You can do probability sampling and non-probability sampling. Simple random sampling, systematic sampling, and stratified random sampling are examples of probability sampling. While purposive sampling and convenience sampling are examples of non-probability sampling.
- The frequency distribution table gives a clear summary of raw data.



Do

Encircle the letter of the correct answer.

1. What refers to the process of selecting individual members from the whole population in which the data is to be gathered?

a. Sample
b. Sampling
c. Data
d. Data gathering

2. Which of the following is not a data-gathering technique?

a. Survey
b. Interview
c. Simple random sampling
d. Registration

3. In a frequency distribution table, what does "frequency" refer to?

a. The type of data being collected
b. The number of times a data value occurs
c. The range of the data
d. The average value of the data

4. Why is random sampling often used in research?

a. It ensures that every item in the population has an equal chance of being selected
b. It allows for easier data collection from the entire population
c. It always produces more accurate results than other sampling methods
d. It is the only recognized method of data collection

5. Which of the following best describes the purpose of a frequency distribution table?
   a. To highlight the largest and smallest data values
   b. To plot the data on a graph or chart
   c. To organize data in a way that shows how often each value occurs
   d. To calculate the mean, median, and mode of a set of data

6. Which statement best describes the relationship between data collection and sampling techniques?
   a. Data collection is a subset of sampling techniques
   b. Sampling techniques dictate the method of data collection
   c. Sampling is always done after data collection
   d. Data collection methods can influence the type of sampling technique used

7. Which of the following examples of data can be collected appropriately and conveniently through registration?
   a. Opponents of Gilas Pilipinas in Fiba World Cup 2023
   b. Effects of using a body soap in washing face
   c. Experience of a crew working at McDonalds
   d. Preferred Milktea flavors of people

8. Which of the following examples of data can be collected appropriately from participants through probability sampling?
   a. Effects of increasing the price of rice to the income of sellers
   b. Effectiveness of using asphalt in repairing a road
   c. Electronics knowledge level of fathers
   d. Alcoholic level of public transportation jeepney drivers

9. Which of the following statements is false?
   a. Data gathering refers to the process of selecting individual members from the whole population in which the data is to be gathered
   b. Survey is a data gathering technique
   c. The data about the experience of a crew working at McDonalds can be collected appropriately and conveniently through surveys
   d. The data about the effects of increasing the price of rice on the income of sellers can be collected appropriately from participants through probability sampling

10. Upon examining a frequency distribution table, you notice that the majority of students scored between 80 to 90 on a test. Which of the following conclusions can you draw from this information?
    a. The test was very difficult
    b. Most students had prepared well for the test
    c. The test was very easy
    d. None of the students studied for the test



Brainstorm

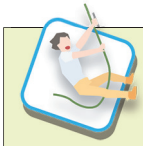
Work in pairs. Your teacher will help you find a partner. Conduct an interview with different people about the data. Ask three different relevant questions.

1. Experience of a person who underwent quarantine during the COVID-19 pandemic
2. Technique of a teacher in teaching mathematics



Design

Work in groups of 4. Your teacher will help you find groupmates. Conduct an experiment to gather the data on the height and weight of your classmates. Construct a frequency distribution table.



Fun Facts

In our modern world, enormous amounts of data are collected every second, shaping how companies and organizations make decisions. Analyzing big data helps in understanding trends and patterns.

LESSON 2

Graphing Data





Objectives

- Use appropriate graphs to represent organized data: pie graph, bar graph, line graph, and stem-and-leaf plot.



Examine

Graphing data is essential in real life. Here are several examples.

The temperatures in different seasons or rainfall patterns over a specific period can be tracked. Population growth or decline of cities or provinces in the Philippines over several years provides a visual representation of demographic changes and helps understand migration patterns. The price changes of common goods or commodities over time can help analyze economic trends and understand how inflation affects everyday expenses. The frequency and types of social media usage in the Philippines can offer insights into preferences and trends related to technology and communication. Which is interesting to you?

Data

- Collecting Data
- Graphing Data
- Interpreting Data



Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).

In this lesson, we will use appropriate graphs to represent organized data. We will be able to visualize our data with the use of these graphs.



Recall

Can you graph how much money you spent per day for the past 7 days? What can you say about your spending habits?

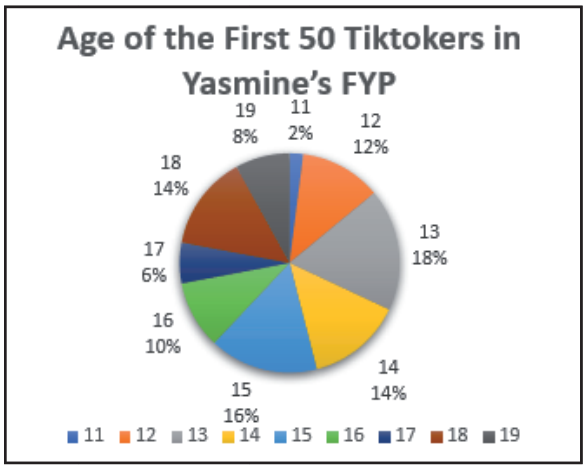


Explore

For this lesson, you will learn about graphical representations for data. First up is the pie graph.

A pie graph is a diagram that is divided into sections to show the frequencies or percentages of different categories within a population or sample. We know that a circle has 360 degrees. To create a pie graph, we need to calculate the angle measure for each category by multiplying the frequency or percentage of that category by 360. Here is an example of a pie graph.

Age	Relative Frequency (rf)	Angle Size (rf*360)
11	0.02	7.2
12	0.12	43.2
13	0.18	64.8
14	0.14	50.4
15	0.16	57.6
16	0.1	36
17	0.06	21.6
18	0.14	50.4
19	0.08	28.8

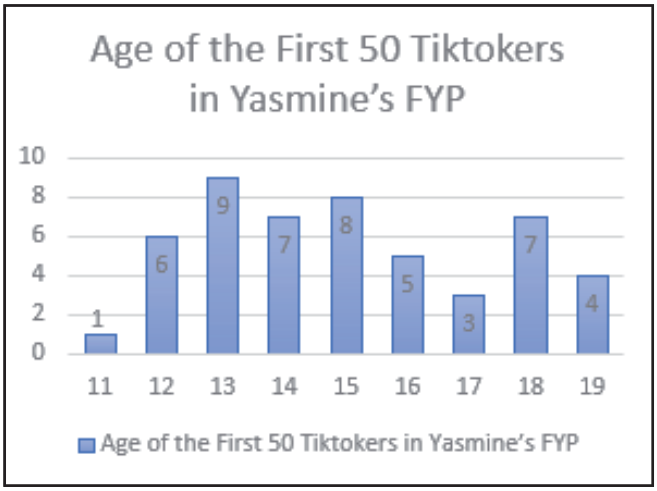


The pie graph shows us a clearer vision of the relationship between the given data.

Next is the bar graph. A bar graph is a graphical representation of data in which rectangular bars are used to represent different categories or groups, and the height or length of each bar corresponds to the frequency, quantity, or value of that category. Bar graphs are commonly used to display categorical data and make it easy to compare different categories visually. Here's how to construct a basic bar graph. Start by identifying the categories or groups you want to represent in your bar graph. These categories should be distinct and non-overlapping. Then, draw a horizontal axis (x-axis) and a vertical axis (y-axis) at right angles

to each other. Label the horizontal axis with the categories you've identified and the vertical axis with the scale for the frequency or values you are representing. Next, determine the appropriate scale for the vertical axis. The scale should cover the entire range of values you want to represent while allowing sufficient space for the bars. Make sure the intervals on the vertical axis are consistent and evenly spaced. Next, for each category, draw a rectangular bar starting from the baseline (usually at zero on the vertical axis) and extending up to the height or length that represents the frequency or value of that category. The width of the bars can be uniform or variable depending on your preference. Next, leave a small gap between adjacent bars to visually separate them. This gap helps in distinguishing between different categories. Next, optionally, you can label each bar with the specific frequency or value it represents. This can make it easier for viewers to interpret the data. Last, don't forget to include a title for your bar graph that describes what the graph is showing. Also, label the axes clearly to indicate what is being measured and the units, if applicable. Bar graphs can be either vertically oriented (categories on the horizontal axis and frequencies on the vertical axis) or horizontally oriented (categories on the vertical axis and frequencies on the horizontal axis). The choice of orientation depends on the data and your preference. Here is an example of a bar graph.

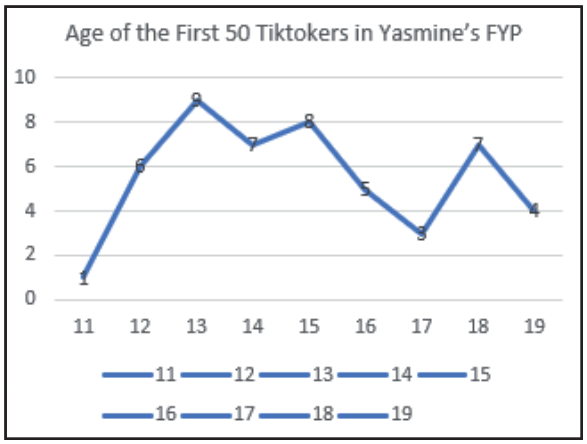
Age	Frequency
11	1
12	6
13	9
14	7
15	8
16	5
17	3
18	7
19	4



Next is the line graph. Line graphs are a common type of chart used to display quantitative values that change over a continuous interval or time span. Here's a breakdown of the key elements and steps involved in creating a line graph. The horizontal axis of a line graph typically represents the independent variable, which could be time or any continuous variable. It consists of labels or values along the continuous scale. The vertical axis represents the dependent variable, which contains the quantitative values that you want to visualize or measure. This axis also consists of a scale to indicate the values. To construct the line graph, you create data points for each specific data value of the dependent variable corresponding to each time or point on the continuous scale. Each data point consists of an x-coordinate (horizontal axis value, e.g., time) and a y-coordinate (vertical axis value, e.g.,

frequency or quantity). After plotting the data points, you connect them sequentially with straight lines. This process creates a continuous line that represents the trend or pattern in the data over the continuous interval. It's essential to label the axes clearly to indicate what is being measured and the units of measurement. Additionally, you should provide a title for the line graph that describes the data, or the relationship being displayed. Line graphs are particularly useful for showing trends, patterns, or changes in data over time or along a continuous scale. The slope of the line can indicate the rate of change, and peaks or valleys in the line can highlight significant points of interest. Here is an example of a line graph.

Age	Frequency
11	1
12	6
13	9
14	7
15	8
16	5
17	3
18	7
19	4



Last is the stem-and-leaf plot. A stem-and-leaf plot is a table wherein each data value is split into a "stem" and "leaf". The stem section is located on the left of the table where the first digit/s are inserted. The leaf section is located on the right of the table where the last digit is placed.

Here is an example. Sam recorded the following temperatures of different places in the Philippines (in degrees Celsius).

16 32 23 36 28 19 35 24 39

Stem	Leaf
1	6 9
2	3 4 8
3	2 5 6 9



Try This

The data below shows the favorite sports of Grade 7 Acceptance. Draw a frequency distribution table and pie graph to organize the raw data.

football basketball volleyball running golf football golf football
golf football football basketball basketball volleyball volleyball running
running basketball volleyball football golf basketball football volleyball
volleyball basketball football football football basketball basketball basketball
volleyball football football football football basketball football basketball

1. Which sport has the lowest frequency?
2. Which sport has the highest frequency?



Key Points

- Graphing the data is a way to present well the data. Unlike the frequency distribution table, the graph presents visually and clearly the trends in data distribution.
- Pie Graph- This is a diagram that is divided into sections to show the frequencies or percentages of different categories within a population or sample.
- Bar Graph- This is a graphical representation of data in which rectangular bars are used to represent different categories or groups, and the height or length of each bar corresponds to the frequency, quantity, or value of that category.
- Line Graph- This is indeed a common type of chart used to display quantitative values that change over a continuous interval or time span.
- Stem-and-Leaf Plot- This is a table wherein each data value is split into a "stem" (1st digit/s) and a "leaf" (last digit).



Do

Encircle the letter of the correct answer.

1. What is a graphical representation of data in which rectangular bars are used to represent different categories or groups, and the height or length of each bar corresponds to the frequency, quantity, or value of that category?

a. Stem-and-Leaf Plot
b. Bar Graph
c. Pie Graph
d. Line Graph

2. What is indeed a common type of chart used to display quantitative values that change over a continuous interval or time span?

a. Stem-and-Leaf Plot
b. Bar Graph
c. Pie Graph
d. Line Graph

3. What is a table wherein each data value is split into a "stem" (1st digit/s) and a "leaf" (last digit)?

a. Stem-and-Leaf Plot
b. Bar Graph
c. Pie Graph
d. Line Graph

4. What does the vertical axis in the bar graph represent?

a. origin
b. categories
c. time
d. frequencies

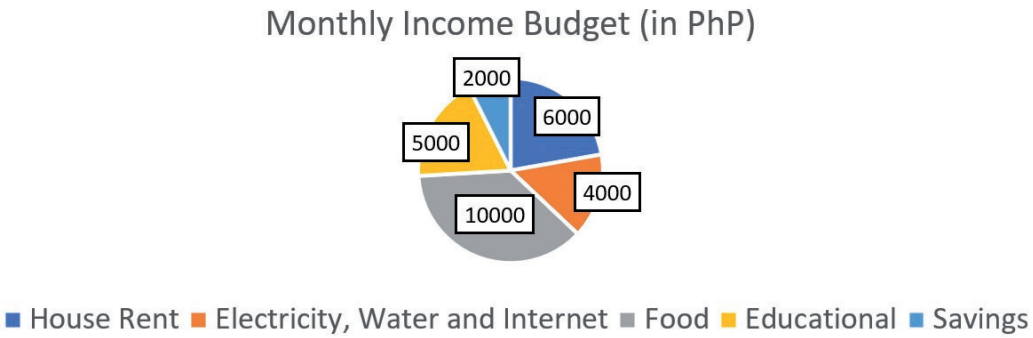
5. What does the vertical axis in the line graph represent?

a. origin
b. frequencies
c. categories
d. time

6. What does the horizontal axis in the line graph represent?

a. origin
b. categories
c. frequencies
d. time

For numbers 7- 8 please use the pie graph:



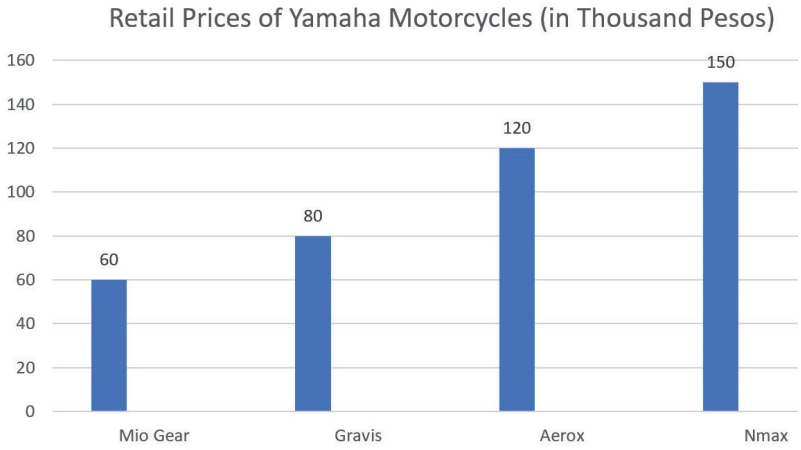
7. How much is the budget for electricity, water, and internet?

a. PhP 4000 b. PhP 6000 c. PhP 10 000 d. PhP 3000

8. How much is the savings of the person from the chart?

a. PhP 6000 b. PhP 5000 c. PhP 4000 d. PhP 2000

For numbers 9- 10 please use the bar graph:



9. Which Yamaha motorcycle is the least expensive from the bar graph?

a. Nmax b. Mio Gear c. Gravis d. Aerox

10. How much is the price of Nmax higher than Mio Gear?

a. PhP 30 000 b. PhP 40 000 c. PhP 60 000 d. PhP 90 000



Brainstorm

Your teacher will group you into pairs. Thirty adults were asked which of the following fried chicken brands they prefer to eat: KFC (K), Jollibee (J), McDonald's (M), Chowking (C), or 7-Eleven (E). Their responses are listed here.

K J M C E M E E K J K M

K C C M J J M J K C C M

J M J K K M

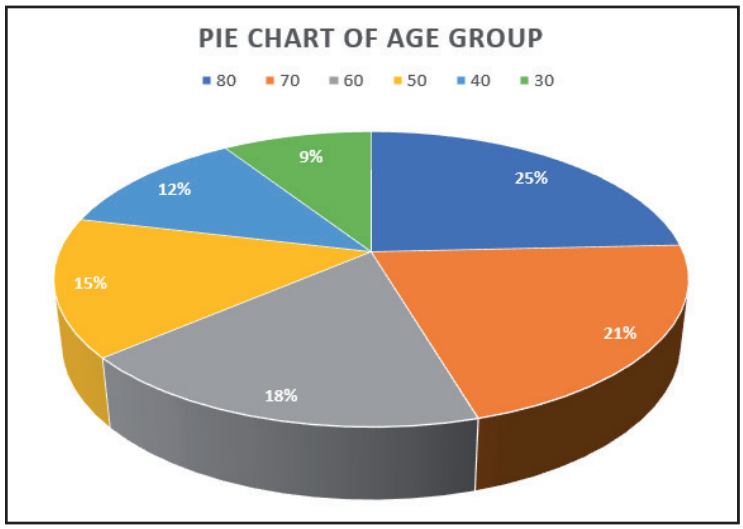
1. Prepare a frequency distribution table.

2. Calculate the relative frequencies and percentages for all categories.

3. Draw a bar graph for the relative frequency distribution.

Design

Your teacher will divide you into groups of 4 members each. Work together to answer the questions. The pie graph shows the data distribution of ages of men recently diagnosed with prostate cancer.



1. Are there more or less than 50% of these patients in their 50s and younger? Explain.
2. Are there more or less than 75% of these patients in their 60s and older? Explain.
3. An interviewer studied the pie chart and stated, "Take a look at these 50-year-old men who are getting prostate cancer. This is a problem for a man once he turns 50." Explain why the interviewer cannot conclude from this pie chart that there are many 50-year-old men with prostate cancer. Can you think of any other way to present this case (both table and graph) to determine if the interviewer's claim is valid?

Fun Facts

The tally chart is probably the simplest of all numeration systems ever used. A tally chart with marks represents numbers. It makes use of single strokes to represent objects that are being counted. One stroke is used for each object.

LESSON 3

Interpreting Data



Objectives

- Interpret statistical graphs.

Examine

After collecting and graphing data, we interpret it. Line graphs representing weather patterns can be interpreted to predict upcoming weather conditions. Analyzing temperature graphs helps predict whether the day will be hot or cold, and analyzing precipitation graphs helps in predicting rain or storms. Graphs showing the population growth of a city or country over several years reveal the rate of growth or decline in a population and the factors influencing it. Analyzing graphs that represent the average daily traffic flow on roads or the occupancy rates on public transportation systems can help commuters understand peak times and plan accordingly.

Data

- Collecting Data
- Graphing Data
- Interpreting Data

Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).

Understanding bar graphs or pie graphs of sales and profits for different products in a business setting help identify which products are selling well and which might need improvement. Which graphs are you excited to interpret?

In this lesson, we will interpret statistical graphs. We will understand the data presented in pie graphs, bar graphs, and line graphs.



Recall

Can you graph how much time you spent studying for the past 2 weeks? What is your interpretation of your study habits?



Explore

In the previous lessons, you learned how to collect, organize, and represent data. In this lesson, you will learn how to interpret the different statistical graphs.

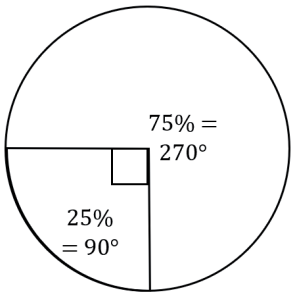
First is the pie graph. A circle can be used to represent 100% and when you divide the circle into fractional parts or sectors, each sector represents a percentage. The sum of these fractional parts or sectors will always equal 100% because the entire circle represents the whole or 100% of something.

This concept is often used to visually represent data in the form of pie graphs, where each sector of the circle corresponds to a different category or data point, and the size of each sector is proportional to the percentage it represents in relation to the whole.

On a pie chart, 25% is equal to $\frac{25}{100}$ times 360 which is 90°.

On a pie chart, 75% is equal to $\frac{75}{100}$ times 360 which is 270°.

A sector with 90° is equal to $\frac{90}{360}$ or 25% of the circle.



Here is an example. Last Saturday, Patricia spent 8 hours a day for working, 3 hours for eating, 6 hours for sleeping, 7 hours for doing hobbies. What percentage of the day was spent on each other?

Adding all the hours of the activities, we get 8 + 3 + 6 + 7 = 24.

We divide the number of hours of each activity by the total hours to get the percentage. Note that some values are rounded off.

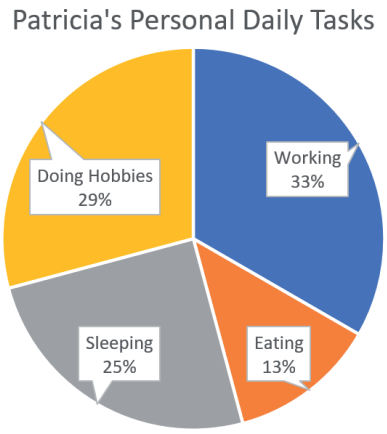
For working, 8 ÷ 24 = 0.33, 0.33(100) = 33%

For sleeping, 6 ÷ 24 = 0.25, 0.25(100) = 25%

For eating, 3 ÷ 24 = 0.13, 0.13(100) = 13%

For hobbies, 7 ÷ 24 = 0.29, 0.29(100) = 29%

Here is the pie graph.

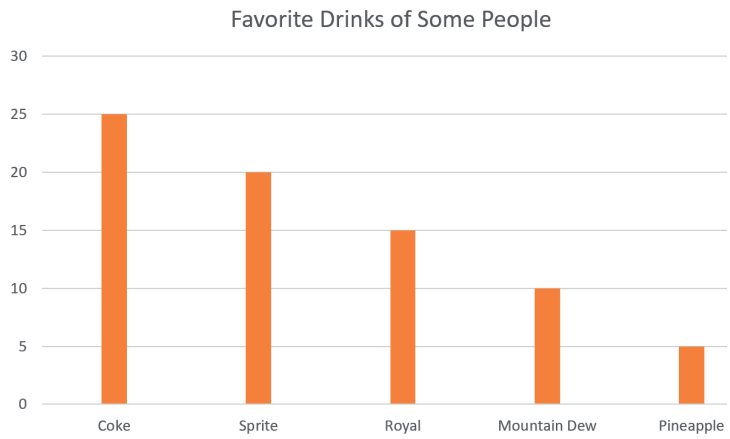


For the interpretation, this pie graph tells us many things like working took up most of the time, eating took up the least amount of time, and more time was spent doing hobbies than sleeping.

Next is the bar graph. In a bar graph, the bars are used to represent quantities and the primary way to make comparisons between these quantities are by examining the lengths of the bars. To ensure accurate visual comparisons, the widths of the bars are typically made equal. Additionally, spacing between the bars is often introduced to separate them from each other and to make them stand out more clearly. This spacing helps prevent visual confusion and allows viewers to easily distinguish between the different bars in the graph. The combination of equal width for the bars and equal spacing between them contributes to the clarity and effectiveness of a bar graph as a tool for visualizing and comparing data.

Here is an example. Eugene surveyed people eating at the fast food to find out their favorite drinks. The result is shown. The data is also represented using a bar graph.

Types of Drinks	Frequency
Coke	25
Sprite	20
Royal	15
Mountain Dew	10
Pineapple Juice	5

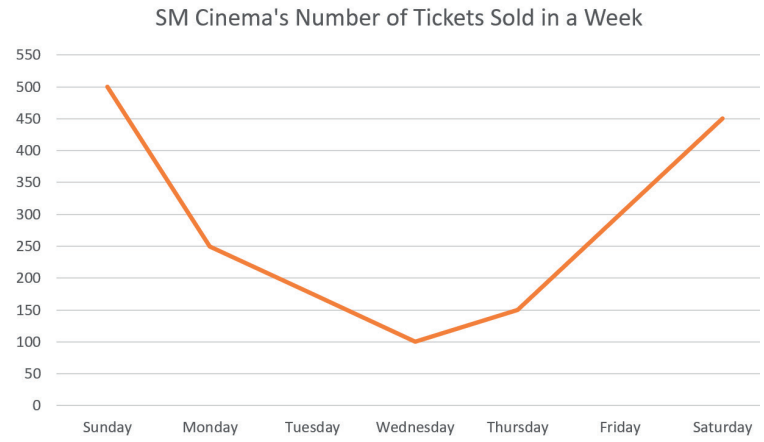


For the interpretation, this bar graph tells us many things like Coke is the favorite drink of these people, pineapple juice is the least favorite, and more people here prefer Sprite than Mountain Dew.

Last is the line graph. Line graphs are a common and effective way to represent data that changes over a continuous range, such as changes in temperature, income, population, and various other variables. Line graphs are valuable tools for visualizing trends and patterns in data, making it easier to understand how one variable (the dependent variable) changes in relation to another (the independent variable) over a continuous range. They are widely used in various fields to analyze and communicate data effectively.

Here is an example. The table below shows the ticket sales of an SM Cinema during a regular week. The line graph follows.

Day	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Number of Tickets Sold	500	250	175	100	150	300	450



For the interpretation, this line graph tells us many things like Wednesday had the fewest number of tickets sold, Sunday had the greatest number of tickets sold, and Monday had more tickets sold than Thursday.



Try This

The table shows the second-month operation expenses of Pie Enterprises.

Categories	Supply	Advertisement	Rental	Salary
Amount	PhP 20 000	PhP 40 000	PhP 60 000	PhP 120 000

Construct a pie graph showing the expenses of Pie Enterprises. Give 3 interpretations from the data.

1. Which category has the greatest expense?
2. Which category has the least expense?



Key Points

- Graphing the data is a way to present well the data. Unlike the frequency distribution table, the graph presents visually and clearly the trends in data distribution.
- A pie graph is a diagram that is divided into sections to show the frequencies or percentages of different categories within a population or sample.
- A bar graph is a graphical representation of data in which rectangular bars are used to represent different categories or groups, and the height or length of each bar corresponds to the frequency, quantity, or value of that category.
- A line graph is indeed a common type of chart used to display quantitative values that change over a continuous interval or time span.
- You can carefully study the different graphs to make correct interpretations about it.



Do

Encircle the letter of the correct answer.

1. What is indeed a common type of chart used to display quantitative values that change over a continuous interval or time span?

   a. Pie Graph
   b. Stem-and-Leaf Plot
   c. Line Graph
   d. Bar Graph

2. What is a diagram that is divided into sections to show the frequencies or percentages of different categories within a population or sample?

   a. Pie Graph
   b. Stem-and-Leaf Plot
   c. Line Graph
   d. Bar Graph

3. What is a graphical representation of data in which rectangular bars are used to represent different categories or groups, and the height or length of each bar corresponds to the frequency, quantity, or value of that category?

a. Pie Graph
b. Stem-and-Leaf Plot
c. Line Graph
d. Bar Graph

4. What does the vertical axis in the line graph represent?

a. origin b. frequencies c. categories d. time

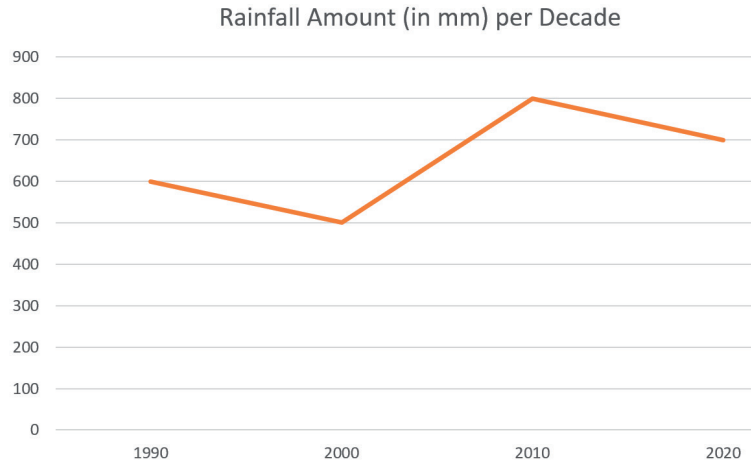
5. What does the horizontal axis in the bar graph represent?

a. origin b. time c. categories d. frequencies

6. What does the horizontal axis in the line graph represent?

a. origin b. time c. categories d. frequencies

For numbers 7- 8 please use the line graph.



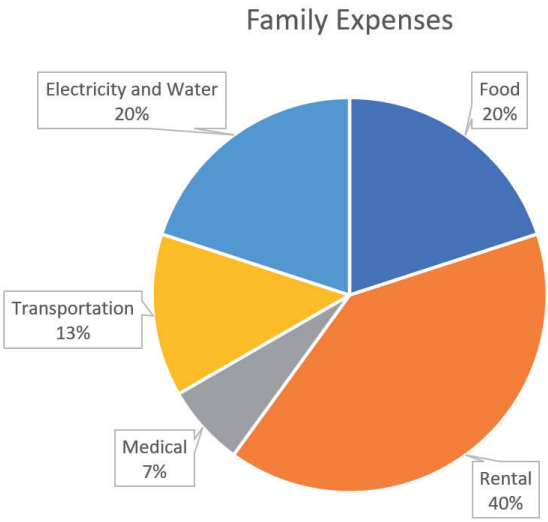
7. Which year has the lowest rainfall amount?

a. 2000 b. 2010 c. 2020 d. 1990

8. How much is the rainfall amount of 2000 lower than 2010?

a. 400 b. 100 c. 200 d. 300

For numbers 9- 10 please use the pie graph.



9. What is the highest expense?

a. Food
b. Transportation
c. Rental
d. Electricity and Water

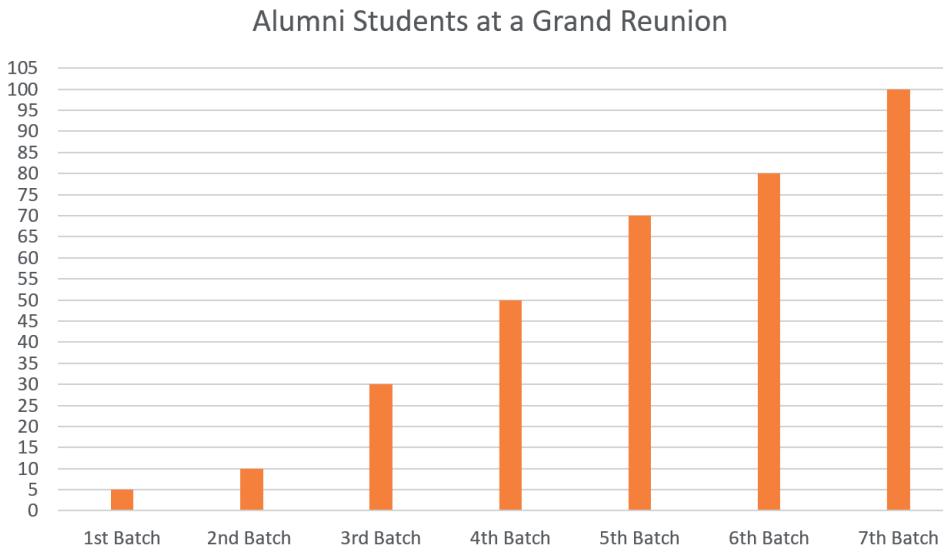
10. If the total amount of family expenses is PhP 15 000, how much will be paid for electricity and water?

a. PhP 3000
b. PhP 4000
c. PhP 5000
d. PhP 6000



Brainstorm

Your teacher will group you into pairs. Use the bar graph to answer the questions.



1. Which batch has the highest number of attendees?

2. Which batch has the lowest number of attendees?

3. How much higher is the number of attendees of the 6th Batch than the 4th Batch?

4. How much higher is the number of attendees of the 5th Batch than the 2nd Batch?

5. How much lower is the number of attendees of the 3rd Batch than the 6th Batch?
6. How much lower is the number of attendees of the 2nd Batch than the 6th Batch?
7. What is the total number of attendees at a grand reunion?



Design

Your teacher will divide you into groups of 4 members each. The table shows the number of enrollees in elementary schools in Region 2 from 2020 to 2023. Draw a line graph to represent the data. Next, each of you will give one interpretation of the data each.

Year	Enrollment (millions)
2020	2.6
2021	3.0
2022	2.1
2023	2.3

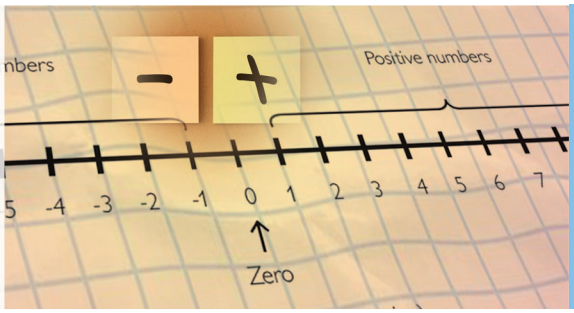


Fun Facts

Sports teams use data to analyze player performance, optimize strategies, and make game-changing decisions. This includes tracking players' movements, shot accuracy, and even predicting opponents' tactics.

LESSON 4

Integers



Objectives

- Describe the set of integers.
- Use positive and negative numbers to describe directions or opposites in real-life situations.
- Locate integers on the number line.

Examine

Integers are used in numerous day-to-day scenarios, helping to represent quantities that have both magnitude and direction.

Integers are used to represent temperature variations. For colder countries, they can have negative numbers in degrees Celsius. Would you want to experience the cold below 0 degrees?

Managing money involves integers. Withdrawals (represented by negative integers) and deposits (represented by positive integers) in a bank account reflect real-life financial operations.

Altitudes and depths are often represented using positive and negative integers. Do you want to ride an airplane flying high or a submarine diving low?

Integers

Integers

Working with Integers

Advanced Operations with Integers

Absolute Value of Integers

Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).

Integers are used to represent distances and directions. The east is represented by positive integers and the west by negative ones.

Sports scores in games like basketball or football often involve positive and negative integers. If a team is leading, it is a positive integer. If the team is trailing, it is a negative integer.

In this lesson, we will describe the set of integers, use positive and negative numbers to describe directions or opposites in real-life situations, and locate integers on the number line.



Recall

Encircle the letter of the correct answer.

1. What is the set of real and rational numbers whose elements are whole numbers and negative counting numbers?

a. Whole Numbers
b. Fractions
c. Radicals
d. Integers

2. What is the set of real and rational numbers whose elements are zero and positive counting numbers?

a. Whole Numbers
b. Fractions
c. Radicals
d. Integers

3. What is the set of real and rational numbers which are not integers?

a. Whole Numbers
b. Radicals
c. Fractions
d. Integers

4. What are the signs of the integers?

a. positive
b. positive and negative
c. negative
d. none of these

5. What does the positive sign in integers indicate?

a. Both insertion or removal of quality or quantity in a variable
b. Not insertion or not removal of quality or quantity in a variable
c. A removal of a quality or quantity in a variable
d. An insertion of a quality or quantity in a variable

6. What is the sign of integers in the right side of a number line composed of positive, zero, and negative integers?

a. no sign
b. positive
c. both positive and negative
d. negative

7. Which of the following is a set of integers?

a. {x|x is a fraction}

b. $\left\{\ldots, -\frac{4}{9}, -\frac{2}{9}, 0, \frac{2}{9}, \frac{4}{9}, \ldots\right\}$

c. {..., -16, -8, -4, 0, 4, 8, 12, ...}

d. $\left\{\ldots, -6\sqrt{3}, -3\sqrt{3}, 0, 3\sqrt{3}, 6\sqrt{3}, \ldots\right\}$

8. Which of the following real-life situations represent a positive integer?

a. Adding 1 cup of water

b. Removing 2 teaspoons of sugar

c. Losing weight of 3 kg

d. Dropping out 7 students in portal

9. What are the integers between -1 and 2?

a. -1, 0, 1, and 2 b. -1 and 2 c. 0 d. 0 and 1

10. How many integers are there between -3 and 2?

a. 4 b. 5 c. 2 d. 3



Explore

Integers are a set of real and rational numbers whose elements are whole numbers and negative counting numbers.

These are examples of a set of whole numbers:

{0, 1, 2} {0, 2, 4, ...} {1, 3, 5, ...}

These are examples of a set of negative counting numbers:

{-1, -2, -3} {-2, -4, -6, ...} {-3, -6, -9, ...}

Hence, these are the examples of a set of integers:

{-1, 0, 1} {..., -2, 0, 2, ...} {..., -3, 1, 5, ...}

The signs of an integer can be used to describe and interpret directions in real-life situations. A positive sign (+) in integers indicates an insertion of a quality or quantity in a variable. A negative sign (-) means that there is a removal of a quality or quantity in a variable.

These are examples of applying integers in describing and interpreting of real-life situations:

1. Adding 1 cup of water = +1

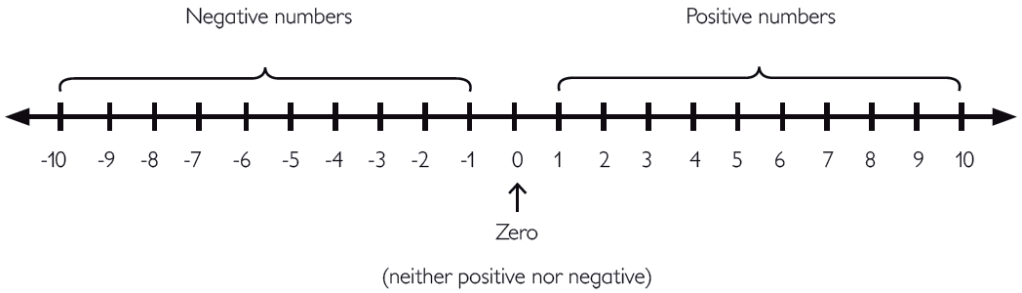
2. Removing two teaspoons of sugar = -2

3. Gaining a height of 3 cm = +3

4. Losing a weight of 4 kg = -4

5. Inserting 5 pages of blank paper = +5

We can visualize the set of integers using a number line.



Here are -4 and +4 plotted on a number line.



These are examples of questions with corresponding answers relevant to the use of number line:

1. What are the integers between -2 and 3? -1, 0, 1, 2
2. How many integers are there between -4 and 3? 6 integers. These are -3, -2, -1, 0, 1, and 2.
3. How many units is 5 from -4? From -4 we count until we reach 5. We traveled 9 units.
4. How many units is -6 from 5? From -6 we count until we reach 5. We traveled 11 units.
5. What is the integer if 7 is added by 3 units? From 7, we count 3 times to the right to get 10.



Try This

Work individually to answer the following questions using the number line.

1. What are the integers between -3 and 4?
2. How many integers are there between -5 and 4?
3. How many units is 6 from -5?
4. How many units is -7 from 6?



Key Points

- Integers are set of real and rational numbers whose elements are whole numbers and negative counting numbers.
- A positive sign (+) in integers indicates an insertion of a quality or quantity in a variable.
- A negative sign (-) means that there is a removal of a quality or quantity in a variable.
- We can visualize the set of integers using a number line.



Do

I. Encircle the letter of the correct answer.

1. What is the set of real and rational numbers whose elements are zero and positive counting numbers?

a. Radicals
b. Fractions
c. Integers
d. Whole Numbers

2. What is the set of real and rational numbers which are not integers?

a. Radicals
b. Fractions
c. Integers
d. Whole Numbers

3. What is the set of real and rational numbers whose elements are whole numbers and negative counting numbers?

a. Radicals
b. Fractions
c. Integers
d. Whole Numbers

4. What are the signs of the integers?

a. positive and negative
b. positive
c. negative
d. none of these

5. What does the negative sign in integers indicate?

a. Both insertion or removal of quality or quantity in a variable
b. Not insertion or not removal of quality or quantity in a variable
c. An insertion of a quality or quantity in a variable
d. A removal of a quality or quantity in a variable

6. What is the sign of integers in the left side of a number line composed of positive, zero, and negative integers?
   a. negative
   b. both positive and negative
   c. positive
   d. no sign

7. Which of the following is a set of integers?
   a. {x|x is a radical}
   b. $\left\{\ldots, -\frac{4}{11}, -\frac{2}{11}, 0, \frac{2}{11}, \frac{4}{11}, \ldots\right\}$
   c. {..., -10, -5, 0, 5, 10, ...}
   d. $\left\{\ldots, -6\sqrt{5}, -3\sqrt{5}, 0, 3\sqrt{5}, 6\sqrt{5}, \ldots\right\}$

8. Which of the following real-life situations represent a negative integer?
   a. Adding 2 cups of water
   b. Removing 3 teaspoons of sugar
   c. Raising the temperature by 4°C
   d. Hiring 5 applicants

9. How many units is 5 from -4?
   a. 6 b. 7 c. 8 d. 9

10. How many units is -6 from 5?
   a. 9 b. 10 c. 11 d. 12

II. Identify where in the number line do the following belong. Write LEFT if it is located on the left side of the value zero otherwise write RIGHT.

1. 9
2. -12
3. 1
4. -2
5. 5
6. -4
7. 70 000
8. -500
9. 1 000 000 000
10. -300 000

III. Use integers to represent the given situations.

1. Increase of PhP 300
2. Going 5 km South
3. Decrease temperature by 5°C
4. Loss of PhP 800 000 in business
5. Moving 30 m to the left
6. Deposit of PhP 1500
7. Withdrawal of PhP 850
8. Profit of PhP 769
9. 3 km downstream
10. Decrease of 56 students in enrollment

IV. Give the opposites of the following.

1. -7
2. 20
3. -16
4. -100
5. 0
6. 18
7. -2
8. 31
9. 3
10. -1

V. Draw the integers on a number line.

1. -1, 0, 1, 2, 3
2. -4, -1, 2, 5, 8
3. 5, -7, -4, -2, 6



Brainstorm

Your teacher will help you find a partner for this pair work. Work together to interpret the following real-life situations by representing them into integers. Put the correct sign before the number.

1. Adding 2 cups of water
2. Losing a weight of 5 kg
3. Inserting 6 pages of blank paper
4. Enrolling 7 students in portal
5. Taking out 9 hamburgers



Design

Your teacher will group you into groups with 4 members each. Work together to determine whether the following sets are sets of integers or not.

1. {..., -5, -2, 1, 4, 7, ...}
2. $\left\{\ldots, -\frac{-3}{7}, \frac{-1}{7}, \frac{1}{7}, \frac{3}{7}, \frac{5}{7}, \ldots\right\}$
3. $\left\{\ldots, -8\sqrt{2}, -3\sqrt{2}, 2\sqrt{2}, 7\sqrt{2}, 12\sqrt{2}, \ldots\right\}$
4. {-1, -8, -15, ...}
5. {x|x is an even number}

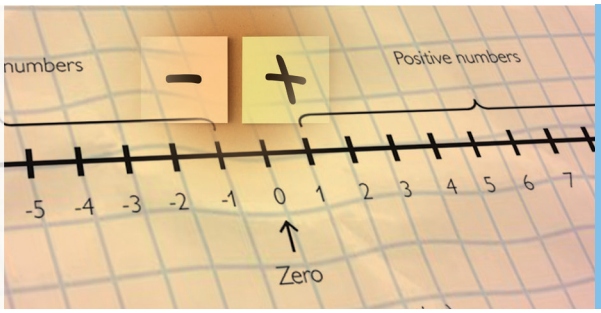


Fun Facts

The word integer came from the Latin word integer which means “untouched” – from in- “not” and the root word of tangere “to touch”. The word was used as an adjective that meant “whole” or “entire”.

LESSON 5

Working with Integers



Objectives

- Compare and order integers.
- Add and subtract integers.

Examine

Integers are used in numerous day-to-day scenarios, helping to represent quantities that have both magnitude and direction.

Integers are used to represent temperature variations. For colder countries, they can have negative numbers in degrees Celsius. Would you want to experience the cold below 0 degrees?

Managing money involves integers. Withdrawals (represented by negative integers) and deposits (represented by positive integers) in a bank account reflect real-life financial operations.

Altitudes and depths are often represented using positive and negative integers. Do you want to ride an airplane flying high or a submarine diving low?

Integers

Integers

Working with Integers

Advanced Operations with Integers

Absolute Value of Integers

Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).

We can compare and order integers in many situations. We can compare different bank account balances, temperature changes, elevation levels, stock market changes, and sports standings. Where have you compared or ordered integers?

We can add and subtract integers in different areas. We can add and subtract integers in financial transactions, temperature changes, elevation changes, gaining or losing points in sports, and in debit and credit. Where have you added or subtracted integers?

In this lesson, we will work with integers by comparing the values of 2 integers and ordering 3 or more integers from least to greatest or vice versa. We will also perform operations of addition and subtraction on integers.



Recall

Encircle the letter of the correct answer.

1. Which of the following is not a sign in comparing integers?

a. + b. > c. < d. =

2. What is the rule in adding two integers with the same sign?

a. Change the subtraction operation into addition as well as the sign of the minuend, then perform the addition operation.
b. Change the subtraction operation into addition as well as the sign of the subtrahend, then perform the addition operation.
c. Subtract the numerical values, then copy the sign of the greater numerical value.
d. Add the numerical values, then copy the common sign.

3. What is the rule in subtracting two integers?

a. Change the subtraction operation into addition as well as the sign of the minuend, then perform the addition operation.
b. Change the subtraction operation into addition as well as the sign of the subtrahend, then perform the addition operation.
c. Subtract the numerical values, then copy the sign of the greater numerical value.
d. Add the numerical values, then copy the common sign.

4. What integers are greater than 0?

a. zero
b. negative integers
c. positive integers
d. none of these

5. What will be the sign of the sum if a negative integer with higher numerical value is added to positive integer with lower numerical value?
   a. negative
   b. positive
   c. zero
   d. none of these
6. What will be the sign of the difference if a positive integer is subtracted by a negative integer?
   a. zero
   b. negative
   c. positive
   d. none of these
7. What is the answer in -6 + -8?
   a. 2
   b. -14
   c. 14
   d. -2
8. What is the answer in 10 + -3?
   a. -7
   b. 13
   c. -13
   d. 7
9. What is the answer in 2 – 12?
   a. -10
   b. 10
   c. -14
   d. 14
10. What is the answer in 11 – (-3)?
    a. -14
    b. 8
    c. -8
    d. 14



Explore

In comparing and ordering integers, signs, and numerical values are being considered. Integers with positive sign (+) are always greater than negative sign (-). Moreover, zero (0) is also an integer that is greater than the negative integers but less than the positive integers.

Here is an example.

The integer 7 is greater than -3 since the sign of the 7 is positive while the -3 is negative.

We write it as 7 > -3.

If the two integers have the same sign, then the numerical values should be observed in comparing and ordering. If both integers are positive, then the one with a higher numerical value is greater than the other.

Here is an example.

The integer 8 is greater than 6 since the 8 is higher numerical value than the 6.

We write it as 8 > 6.

We can also order 3 integers from least to greatest. This is called in ascending order. If the integers 5, -3, and 9 are arranged in ascending order, then the result will be -3, 5, and 9.

If both integers are negative, then the one with a lower numerical value is greater than the other.

Here is another example.

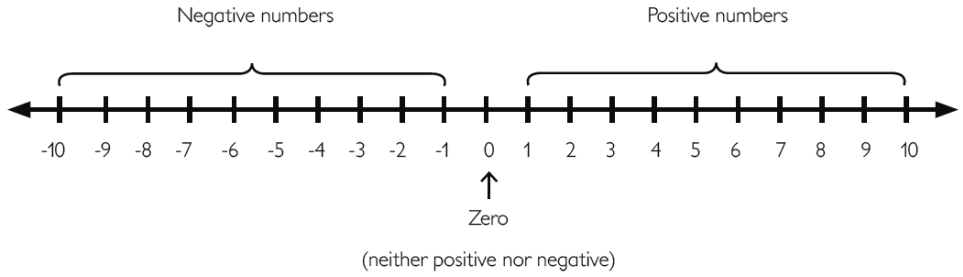
The integer -2 is greater than -10 since the 2 is a lower numerical value than the 10.

We write it as -2 > -10.

Here is another example in ordering 3 integers. This time, it is from greatest to least. This is called in descending order.

If the integers 3, -11, and -7 are arranged in descending order, then the result will be 3, -7, and -11.

The number line can be used to compare and order integers. Integers located at the right side of a number line are greater than those who located at the left side.



We can perform operations on integers. Two integers with the same signs can be added. First, add the numerical values. Then, copy the common sign.

Here are some examples.

The answer in 5 + 2 is equal to 7. Adding the numerical values 5 and 2, the result will be 7. Copying the signs of 5 and 2 which are both positive, the result will be 7.

The answer in -4 + -6 is equal to -10. Adding the numerical values 4 and 6, the result will be 10. Copying the signs of -4 and -6 which are both negative, the result will be -10.

Two integers with different signs can also be added. First, subtract the numerical values. Then, copy the sign of the integer with greater numerical value.

Here are some examples.

The answer in 8 + -3 is equal to 5. Subtracting the numerical values 8 and 3, the result will be 5. Copying the sign of the greater numerical value 8 which is positive, the result will be 5.

The answer in -9 + 2 is equal to -7. Subtracting the numerical values -9 and 2, the result will be 7. Copying the sign of the greater numerical value -9 which is negative, the result will be -7.

Aside from addition, two integers can also be subtracted. First, change the subtraction operation into addition. Then, change the sign of the subtrahend. Next, perform the addition operation.

Here are some examples.

The answer in 2 – 5 is equal to -3. Changing the subtraction sign into the addition sign, the result will be 2 + 5. Changing the sign of the subtrahend, the result will be 2 + -5. Solving 2 + -5, the result will be -3.

The answer in 6 – (-3) is equal to 9. Changing the subtraction sign into the addition sign, the result will be 6 + -3. Changing the sign of the subtrahend, the result will be 6 + 3. Solving 6 + 3, the result will be 9.



Try This

Work individually to solve the following items.

1. 9 – 5 =

2. 6 – (-2) =

3. -8 – 6 =

4. -3 – (-5) =



Key Points

- In comparing and ordering integers, signs, and numerical values are being considered. Integers with positive sign (+) are always greater than negative sign (-). Moreover, zero (0) is also an integer that is greater than the negative integers but less than the positive integers.
- Two integers with the same signs can be added. First, add the numerical values. Then, copy the common sign. Two integers with different signs can also be added. First, subtract the numerical values. Then, copy the sign of the integer with greater numerical value.
- Aside from addition, two integers can also be subtracted. First, change the subtraction operation into addition. Then, change the sign of the subtrahend. Next, perform the addition operation.



Do

I. Encircle the letter of the correct answer.

1. Which of the following is not a sign in comparing integers?

a. + b. > c. < d. =

2. What is the rule in adding two integers with the different sign?

a. Change the subtraction operation into addition as well as the sign of the minuend, then perform the addition operation.
b. Change the subtraction operation into addition as well as the sign of the subtrahend, then perform the addition operation.
c. Subtract the numerical values, then copy the sign of the greater numerical value.
d. Add the numerical values, then copy the common sign.

3. What is the rule in subtracting two integers?

a. Change the subtraction operation into addition as well as the sign of the minuend, then perform the addition operation.
b. Change the subtraction operation into addition as well as the sign of the subtrahend, then perform the addition operation.
c. Subtract the numerical values, then copy the sign of the greater numerical value.
d. Add the numerical values, then copy the common sign.

4. Which of the following shows the arrangement of integers in ascending order?

a. negative, positive, zero
b. positive, zero, negative
c. positive, negative, zero
d. negative, zero, positive

5. What will be the sign of the sum if a negative integer with lower numerical value is added to positive integer with higher numerical value?

a. positive b. negative c. zero d. none of these

6. What will be the sign of the difference if a negative integer is subtracted by a positive integer?

a. one b. zero c. positive d. negative

7. What is the answer in -5 + -12?

a. 7 b. -17 c. 17 d. -7

8. What is the answer in -15 + 3?

a. -18 b. 12 c. -12 d. 18

9. What is the answer in 6 – 7?

a. -1 b. 1 c. -13 d. 13

10. What is the answer in -15 – 5?

a. -10 b. 10 c. -20 d. 20

II. Answer the following questions.

1. 5 + 1 =

2. -5 - 1 =

3. 6 + 3 =

4. -6 + 3 =

5. -6 - 3 =

6. 3 - (-19) =

7. -15 - 25 =

8. -89 - (-27) =

9. 17 - 43 =

III. Compare the following integers. Write > or <.

1. -5 ___ 5

2. 0 ___ -2

3. -6 ___ -1

4. -4 ___ -7

5. -8 ___ 0

6. -27 ___ 0

7. -145 ___ -143

8. -1000 ___ 10

9. -2018 ___ -2017

IV. Arrange the following integers according to the given order.

1. {-1, 0, -8, -6, -2} : increasing order

2. {10, -15, 25, -25, 30} : decreasing order

3. {-5, 5, 0, 1, -1} : increasing order

V. Read the statements for each number. Write:

M if statement A alone is enough to answer the question.
A if statement B alone is enough to answer the question.
T if you need both statements to answer the question.
H if statements A and B together are not enough to answer the question.

1. A. An order for 253 bracelets is placed.
B. There are 411 bracelets ready to be sold.

QUESTION: How many bracelets are left in the stocks after the order was delivered?

2. A. The sum of two integers is positive.
B. One of the integers is positive.

QUESTION: What is the sign of the other integer?

3. A. The sum of two integers is -85.
   B. One of the integers is positive.

   QUESTION: What is the sign of the other integer?

4. A. The subtrahend is a positive integer.
   B. The minuend is a negative integer.

   QUESTION: Is the difference a positive integer, negative integer, or zero?

5. A. Ryan's sales were greater than his expenses.
   B. Ryan's total expense was PhP 17.

   QUESTION: Did Ryan get a profit or a loss?

VI. Solve the following problems.

1. A shop had 320 boxes of pencils in stock. A truck delivered an additional of 570 boxes to the shop's supply. In a week, the shop had sold 418 boxes of pencils. How many boxes of pencils were left at the end of the week?
2. At 5 am, the temperature was 24°C. At around noon, it rose by 5°C and went down 7°C early in the evening at 6:30 pm. What was the temperature at 6:30 pm?
3. In golf, the term par indicates the average number of shots a good golf player takes to sink the ball into the hole. A player's score is the number you add to the par in order to get the number of shots a player took for a certain hole. On a par 5 hole, if a player's score was -2, how many shots did he take?
4. The sum of four consecutive integers is -2. What is the smallest of the four?



Brainstorm

Your teacher will help you find a partner. Work together to solve the following items.

1. 9 + 6 =
2. -6 + -9 =
3. 7 + -3 =
4. -6 + 9 =



Design

Your teacher will group you into groups of 4 members each. Work together to arrange the following integers in ascending order.

1. -5, -10, 4

2. 8, -3, -5, 9

3. 6, -2, 9, -6, 8

4. -7, 15, -2, 8, -6, 9

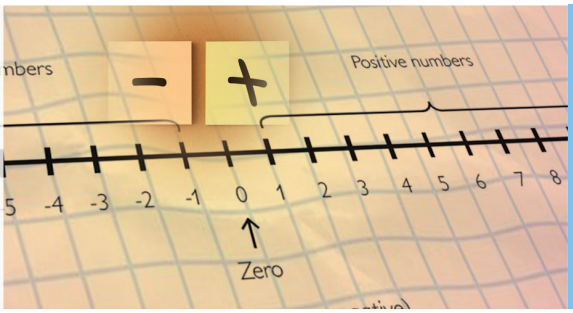


Fun Facts

The lowest temperature ever recorded is -128.6°F in Antarctica, while the highest recorded temperature is +134°F in California's Death Valley.

LESSON 6

Advanced Operations with Integers



Objectives

- Multiply and divide integers.
- Simplify numerical expressions involving integers using number properties and the order of operations.

Examine

Multiplication and division of integers have practical applications beyond mathematical problems. In accounting, multiplying and dividing integers come into play when calculating financial debts and credits. For instance, if a person owes money and they need to pay back multiple installments, this involves multiplying negative integers. Companies use multiplying and dividing integers when calculating profits and losses. If a business gains a profit that's a certain percentage of their initial investment or incurs a loss, it involves these operations. What business would you want in the future?

Integers

Integers

Working with Integers

Advanced Operations with Integers

Absolute Value of Integers

Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).

When discussing temperature, multiplying and dividing integers can be seen in temperature changes. For instance, if the temperature drops by 3 degrees Celsius per hour, this demonstrates multiplying by a negative integer. Population changes can illustrate multiplying and dividing integers. When a population grows at a certain rate annually or declines by a percentage, it involves multiplying and dividing integers.

In sports, scores or rankings can involve multiplying and dividing integers. If an athlete's performance improves by a certain percentage or declines by a fraction, it involves these operations. What is your favorite sport?

In this lesson, we will perform advanced operations by multiplying and dividing integers, and simplify numerical expressions involving integers using number properties and the order of operations.



Recall

Encircle the letter of the correct answer.

1. What is the rule in multiplying two integers with the same signs?

a. Multiply the numerical values, then insert a negative sign.
b. Multiply the numerical values, then insert a positive sign.
c. Divide the numerical value of the dividend by the divisor, then insert a negative sign.
d. Divide the numerical value of the dividend by the divisor, then insert a positive sign.

2. What is the rule in dividing two integers with different signs?

a. Multiply the numerical values, then insert a negative sign.
b. Multiply the numerical values, then insert a positive sign.
c. Divide the numerical value of the dividend by the divisor, then insert a negative sign.
d. Divide the numerical value of the dividend by the divisor, then insert a positive sign.

3. What does the letter G mean from the word GEMDAS?

a. Grouping
b. Greatest Common Factor
c. Geometry
d. Greek

4. What is the sign of the product if a negative integer is multiplied by another negative integer?

a. zero
b. negative
c. none of these
d. positive

5. What will be the sign of the product if a negative integer is divided by a positive integer?

a. zero
b. negative
c. none of these
d. positive

6. The operations of an expression, from left to right order, are addition, subtraction, and multiplication. Which operation should be performed first?

a. addition b. subtraction c. none of these d. multiplication

7. What is the answer to $9(7)$?

a. 63 b. -63 c. 56 d. -56

8. What is the answer to $\frac{-48}{8}$?

a. -7 b. 7 c. -6 d. 6

9. What is the answer to $2 + 3 - 10$?

a. -9 b. -5 c. 9 d. 5

10. What is the answer to $\frac{-4(5)}{2}$?

a. -20 b. 20 c. -10 d. 10



Explore

Two integers with the same signs can be multiplied. First, multiply the numerical values. The product is positive.

Here are some examples.

The answer to $5(4)$ is 20. Multiplying the numerical values 5 and 4, the result is 20. The product is positive, so the result is 20.

The answer to $-6(-3)$ is 18. Multiplying the numerical values 6 and 3, the result is 18. The product is positive, so the result is 18.

Two integers with the different signs can be multiplied. First, multiply the numerical values. Then, insert the negative sign as the product is negative.

Here are some examples.

The answer to $-7(4)$ is -28. Multiplying the numerical values 7 and 4, the result is 28. Inserting the negative sign, the result is -28.

The answer to $5(-8)$ is -40. Multiplying the numerical values 5 and 8, the result is 40. Inserting the negative sign, the result is -40.

Two integers with the same signs can be divided. First, divide the numerical value of the dividend by the divisor. The quotient is positive.

Here are some examples.

The answer to 24 ÷ 6 is 4. Dividing the numerical value 24 by 6, the result is 4. The quotient is positive, so the result is 4.

The answer to -35 ÷ -7 is 5. Dividing the numerical value 35 by 7, the result is 5. The quotient is positive, so the result is 5.

Two integers with the different signs can be divided. First, divide the numerical value of the dividend by the divisor. Then, insert the negative sign as the quotient is negative.

Here are some examples.

The answer to -56 ÷ 8 is -7. Dividing the numerical value 56 by 8, the result is 7. Inserting the negative sign, the result is -7.

The answer to 72 ÷ -9 is -8. Dividing the numerical value 72 by 9, the result is 8. Inserting the negative sign, the result is -8.

In solving the expressions involving more than two integers and more than one operation, the GEMDAS rule should be applied. Each letter of the word GEMDAS should be applied in order. The "G" in GEMDAS stands for grouping which includes braces, brackets, and parenthesis. Here, expressions inside the braces should be solved first. Then, it is followed solving expressions inside the brackets. The next one is solving expressions inside the parentheses. The "E" in GEMDAS stands for exponent. After solving all expressions inside the grouping symbols, this is followed by solving the exponents. The "MD" in GEMDAS stands for multiplication or division. From the left of the expression to the right, all multiplication or division operations should be solved. Last, the "AS" in GEMDAS stands for addition or subtraction. Similar with "MD", all addition or subtraction operations should be solved from the left of the expression to the right.

Here are some examples.

$$3 + 7(6) - 2$$

Applying first multiplication in the expression, the result is 3 + 42 – 2.

Applying addition in the expression, the result is 45 – 2.

Finally applying subtraction, the result is 43.

$$3(2 - 8) + 10$$

Applying first grouping in the expression, the result after subtracting inside the parenthesis is 3(-6) + 10.

Applying multiplication in the expression, the result is -18 + 10.

Applying addition in the expression, the result is -8.

$$\frac{(12-10)^3}{-4}$$

Applying first grouping in the expression, the result is $\frac{2^3}{-4}$.

Applying exponent in the expression, the result is $\frac{8}{-4}$.

Applying division in the expression, the result is -2.



Try This

Work individually to solve the following items.

1. 6(9) =
2. -5(-10) =
3. -4(8) =
4. 6(-7) =
5. 45 ÷ 5 =
6. -48 ÷ -6 =
7. -54 ÷ 9 =
8. 49 ÷ -7 =



Key Points

- Two integers with the same signs can be multiplied. First, multiply the numerical values. The product is positive. Two integers with the different signs can be multiplied. First, multiply the numerical values. Then, insert the negative sign.
- Two integers with the same signs can be divided. First, divide the numerical value of the dividend by the divisor. The quotient is positive. Two integers with the different signs can be divided. First, divide the numerical value of the dividend by the divisor. Then, insert the negative sign.
- In solving expressions involving more than two integers and more than one operation, the GEMDAS rule should be applied. Each letter of the word GEMDAS should be applied in order. The "G" in GEMDAS stands for grouping which includes braces, brackets, and parentheses. The "E" in GEMDAS stands for exponent. The "MD" in GEMDAS stands for multiplication or division. Last, the "AS" in GEMDAS stands for addition or subtraction.



Do

I. Encircle the letter of the correct answer.

1. What is the rule in multiplying two integers with different signs?

a. Multiply the numerical values, then insert a positive sign.
b. Multiply the numerical values, then insert a negative sign.
c. Divide the numerical value of the dividend by the divisor, then insert a negative sign.
d. Divide the numerical value of the dividend by the divisor, then insert a positive sign.

2. What is the rule in dividing two integers with the same signs?

a. Multiply the numerical values, then insert a positive sign.
b. Multiply the numerical values, then insert a negative sign.
c. Divide the numerical value of the dividend by the divisor, then insert a negative sign.
d. Divide the numerical value of the dividend by the divisor, then insert a positive sign.

3. What does the letter E mean from the word GEMDAS?

a. Exponents b. Extremes c. Equations d. Expressions

4. What will be the sign of the product if a positive integer is multiplied by another positive integer?

a. zero b. negative c. positive d. none of these

5. What will be the sign of the product if a positive integer is divided by a negative integer?

a. zero b. negative c. positive d. none of these

6. The operations of an expression, from left to right order, are addition, subtraction, and multiplication. Which operation should be performed last?

a. multiplication b. addition c. subtraction d. none of these

7. What is the answer to $8(-9)$?

a. -72 b. 72 c. -81 d. 81

8. What is the answer to $\frac{-56}{-7}$?

a. -9 b. 9 c. -8 d. 8

9. What is the answer to $3 - 6 + 7$?

a. -10 b. 4 c. -4 d. 10

10. What is the answer in $\frac{-6(4)}{3}$?

a. 6 b. -6 c. 8 d. -8

II. Answer the following questions.

1. -15(-1) =
2. -15 ÷ -1 =
3. 12 ÷ -6 =
4. 16(-3) =
5. -13(-6) =
6. 8(-14) =
7. -16(-30) =
8. -19(25) =
9. -81 ÷ (-27) =
10. 63 ÷ (-7) =
11. -430 ÷ 5 =

III. Solve the following problems.

1. A scuba diver is at a depth of 32 m. This can be represented by the integer -32. What integer can represent the depth of another diver at a depth three times as that of the first diver?
2. A mountaineer was at the top of Mount Pulag. He spent 5 hours going 480 m down the mountain. Assuming he went on a constant pace, what integer would represent his change in elevation per hour?
3. The oven temperature dropped 525°F in 15 minutes. What integer represents the number of degrees per minute the temperature dropped?
4. A mountaineer is at the top of Mount Apo. He spends 5 hours going 525 m down the mountain. Assuming he went on a constant pace, what integer would represent his change in elevation per hour?
5. A scuba diver is at a depth of 45 m which could be represented by the integer -45. What integer would represent the depth of another diver at a depth twice that of the first diver?



Brainstorm

Your teacher will find you a partner. Work together to solve the following items.

1. 12 + 4 – 12
2. $\frac{4(-6)}{2}$
3. 5 + 3(-2) – 10
4. $\frac{-2(9)}{3} + 5$



Design

Your teacher will divide you into groups of 4 members each. Work together to solve the following items.

1. $-2(3-5)+5$ **2.** $\frac{-6(7-3)}{2}$ **3.** $2^5+3(4-6)$ **4.** $\frac{9(8-5)^4}{3}$

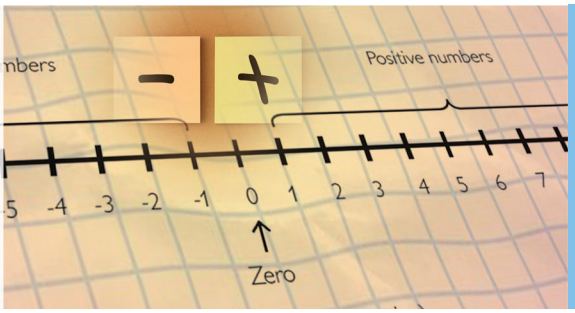


Fun Facts

Any number multiplied by zero results in zero. This applies to integers as well. Regardless of the sign (+ or -), when an integer is multiplied by zero, the product will always be zero.

LESSON 7

Absolute Value of Integers





Objectives

- Identify the absolute value of an integer.
- Understand the absolute value of an integer on the number line.



Examine

We have discussed much about integers. We will now discuss about their absolute values. Here are some real-life situations.

Understanding the absolute value of temperatures can be crucial. For instance, if the temperature drops from 10°C to -5°C, the absolute value of the change is 15°C, irrespective of the direction (increase or decrease).

When navigating a map, considering distances traveled or directions from home, absolute value comes into play. If a person walks 8 km north or 8 km south from their starting point, the absolute value of their displacement from home is 8 km. What is the farthest distance you walked?

Considering the absolute value of speed or velocity in different directions is crucial in physics or transportation. For example, a car traveling at 60 km/h east or 60 km/h west has an absolute velocity of 60 km/h.

Integers

Integers

Working with Integers

Advanced Operations with Integers

Absolute Value of Integers



Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).

In geography or while climbing, understanding the absolute value of elevation changes is essential. If a mountain climber ascends 1000 meters or descends 1000 meters, the absolute value of their net change in altitude is 1000 meters. Have you ever climbed a mountain?

Suppose someone withdraws PhP 500 from their bank account, and someone else then deposits PhP 500. Despite the withdrawal and deposit being opposite actions, the absolute value of the net change in their account balance remains PhP 500.

In this lesson, we will identify the absolute value of an integer and its meaning on the number line.



Recall

Encircle the letter of the correct answer.

1. What is the distance of an integer from zero in the number line?

a. Absolute zero
b. Absolute one
c. Absolute value of an integer
d. Mean absolute deviation

2. Which of the following is the representation of the absolute value of y?

a. {y} b. |y| c. (y) d. [y]

3. What is the sign of the absolute value of any integer?

a. None of these b. Zero c. Negative d. Positive

4. What will be the sign of the absolute value of the sum between two negative integers?

a. Positive b. Negative c. Zero d. None of these

5. What will be the sign of the absolute value of the difference between two positive integers?

a. Zero b. Negative c. Positive d. None of these

6. What will be the sign of the answer if a negative integer is multiplied by the absolute value of a negative integer?

a. Negative b. Positive c. Zero d. None of these

7. What is |-3|?

a. -13 b. 13 c. -3 d. 3

8. What is the arrangement of |-5|, -6, 8 in ascending order?

a. |-5|, -6, 8 b. -6, |-5|, 8 c. -6, 8, |-5| d. |-5|, 8, -6

9. What is |-7 + -9|?

a. 2 b. -2 c. 16 d. -16

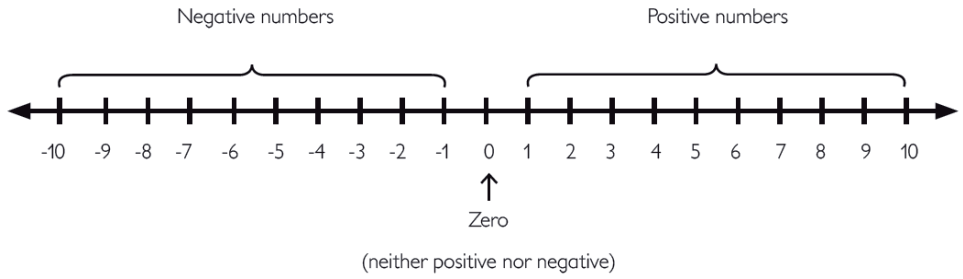
10. What is |6 – 11|?

a. -5 b. 5 c. -17 d. 17



Explore

The absolute value of an integer is the distance of it from zero in the number line. Since the absolute value refers to the distance, then it always has a positive sign. Hence, the number line can be used to find the absolute value of an integer. Moreover, if x represents an integer, then |x| represents the absolute value of an integer.



Here are some examples.

What is |5|?

The distance between 5 and 0 in the number line is 5 units. Therefore, the absolute of 5 is 5.

What is |-4|?

The distance between -4 and 0 in the number line is 4 units. Therefore, the absolute of -4 is 4.

The absolute value can also be integrated in comparing and ordering integers.

Here are some examples.

Which is greater between |-10| and 6?

The absolute value of -10 is 10 which is greater than 6. Therefore, the answer is |-10|.

We write it as |-10| > 6.

Arrange 12, -6, |-3| in ascending order.

The |-3| is 3 which is greater than -6 but less than 12. Therefore, the answer is -6, |-3|, 12.

In adding integers, the absolute value can also be integrated.

Here are some examples.

What is the answer to |-6 + -8|?

Solving the expression inside the absolute value symbol, the result is |-14|. Evaluating the |-14|, the result is 14.

What is the answer to |-7| + 10?

Evaluating the |-7|, the result is 7. Hence, |-7| + 10 becomes 7 + 10. Solving 7 + 10, the result is 17.

Like adding integers, the absolute value can be applied in subtracting integers.

Here are some examples.

What is the answer to |2 – 10|?

Solving the expression inside the absolute value symbol, the result is |-8|. Evaluating |-8|, the result is 8.

What is the answer to 5 – |-9|?

Evaluating |-9|, the result is 9. Hence, 5 – |-9| becomes 5 – 9. Solving 5 – 9, the result is -4.

The absolute value can also be applied in multiplying integers.

Here are some examples.

What is the answer to |-6(-5)|?

Solving the expression inside the absolute value symbol, the result is |30|. Evaluating |30|, the result is 30.

What is the answer to 4(|-7|)?

Evaluating |-7|, the result is 7. Hence, 4(|-7|) becomes 4(7). Solving 4(7), the result is 28.

In dividing integers, there are instances in which the absolute value of the dividend or divisor is to be found.

Here are some examples.

What is the answer to $\left|\frac{-42}{-6}\right|$?

Solving the expression inside the absolute value symbol, the result is |7|. Evaluating |7|, the result is 7.

What is the answer to $\frac{|-56|}{7}$?

Evaluating |-56|, the result is 56. Hence, $\frac{|-56|}{7}$ becomes $\frac{56}{7}$. Solving $\frac{56}{7}$, the result is 8.

Even in applying the GEMDAS rule to solve expressions, the absolute value can also be integrated.

Here are some examples.

What is the answer to |-6 + 11(-3)|?

Applying first multiplication in the expression, the result is |-6 + -33|.

Applying addition in the expression, the result is |-39|.

Evaluating |-39|, the result is 39.

What is the answer to $\left|\frac{-32}{8}\right| - 7$?

Applying first division in the expression, the result is |-4| - 7.

Evaluating |-4|, the result is 4.

Hence, the will become 4 – 7.

Solving 4 – 7, the result is -3.



Try This

Work individually to answer the following items.

1. What is |-26|?
2. Which is greater; |-15| or 9?
3. Which is lesser; |-8| or -3?
4. Arrange 3, |-6|, -4 in ascending order.
5. Arrange -5, |-6|, -2, |8| in descending order.



Key Points

- The absolute value of an integer is the distance of it from zero in the number line.
- The absolute value can also be integrated in comparing and ordering, and the operations on integers.



Do

I. Encircle the letter of the correct answer.

1. What is the distance of an integer from zero in the number line?

a. Absolute zero
b. Absolute one
c. Absolute value of an integer
d. Mean absolute deviation

2. Which of the following is the representation of the absolute value of z?

a. |z|
b. (z)
c. [z]
d. {z}

3. What is the sign of the absolute value of any integer?

a. None of these
b. Zero
c. Negative
d. Positive

4. What will be the sign of the absolute value of the sum between two negative integers?

a. Zero
b. Positive
c. Negative
d. None of these

5. What will be the sign of the absolute value of the difference between two positive integers?

a. Zero
b. Negative
c. Positive
d. None of these

6. What will be the sign of the answer if a negative integer is multiplied by the absolute value of a negative integer?

a. Positive
b. Negative
c. Zero
d. None of these

7. What is |-2|?

a. -12
b. 12
c. -2
d. 2

8. What is the arrangement of 7, -5, |-4| in ascending order?

a. -5, |-4|, 7
b. -5, 7, |-4|
c. |-4|, -5, 7
d. |-4|, 7, -5

9. What is |-4 + 14|?

a. 18 b. -18 c. 10 d. -10

10. What is |3 – (-6)|?

a. 9 b. -9 c. 3 d. -3

II. Give the absolute value of the following integers.

1. 5
2. -5
3. 2
4. -2
5. 10
6. -10
7. 0

III. Represent the absolute value of the following integers on the number line.

1. -4 **2.** 3 **3.** -6

IV. Compare the following. Write >, <, or =.

1. 5 ___ |-6|
2. |7| ___ -8
3. |4| ___ |-4|
4. -2 ___ |-3|
5. |1| ___ |9|
6. |-10| ___ |-11|
7. |-100| ___ |200|
8. |50| ___ 25

V. Answer the following questions.

1. |15 + 1| =
2. |-15 - 1| =
3. |-15(-1)| =
4. |-15 ÷ -1| =
5. |16 + 3| =
6. |-16 + 3| =
7. |-16 - 3| =
8. |16(-3)| =
9. |-13(-6)| =
10. |12 ÷ -6| =



Brainstorm

Your teacher will find you a partner. Work together to solve the following items.

1. |-8 + -5| **2.** |3 – 11| **3.** |-6(-8)| **4.** $\left|\frac{-64}{-8}\right|$



Design

Your teacher will divide you into groups of 4 members each. Work together to solve the following items.

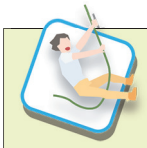
1. $|-4(12) - 5|$

2. $6 + \left|\frac{-54}{9}\right|$

3. $|(-2)^3| + 3(-10)$

4. $|-3^2| + \frac{48}{6}$

5. $4|-6 - 5| - 4^2$



Fun Facts

The absolute value of a number reflects its distance from zero on a number line. Interestingly, it's like a mirror image; whether a number is positive or negative, its absolute value is always positive.

Unit Performance Task: Community Survey and Integer Workbook

Your teacher will group you into teams of 4 members each. Work together to do the following.

Component 1: Statistical Data Collection and Analysis

Conduct a community data collection survey using the skills you learned in English class on Philippine religious practices (e.g., what you learned from Filipino class- *pasyon*, *senakulo*, *Flores de Mayo*)

Use different sampling techniques to collect data.

Organize data into a frequency distribution table.

Represent the collected data using pie graphs, bar graphs, line graphs, or stem-and-leaf plots.

Write a report interpreting the data and graphs, including insights and trends observed.

Component 2: Integer Concept Application

Create an integer workbook.

Describe integers and their use in real-world contexts using what you learned into Science class, like displacement and velocity.

Include 4 exercises on locating, comparing, and ordering integers on a number line.

Develop 4 problems and solutions each for adding, subtracting, multiplying, and dividing integers using different models.

Explain and give 4 examples of the absolute value of integers and their representation on the number line.

Include a section on simplifying 4 numerical expressions involving integers using the order of operations (GEMDAS).

Class Presentation

Groups will present their findings from the community data collection, explaining their methods, data organization, and graph interpretations.

Demonstrate understanding of integer concepts through examples from the integer workbook.

Each presentation should include visual aids (graphs, number lines) and be interactive.

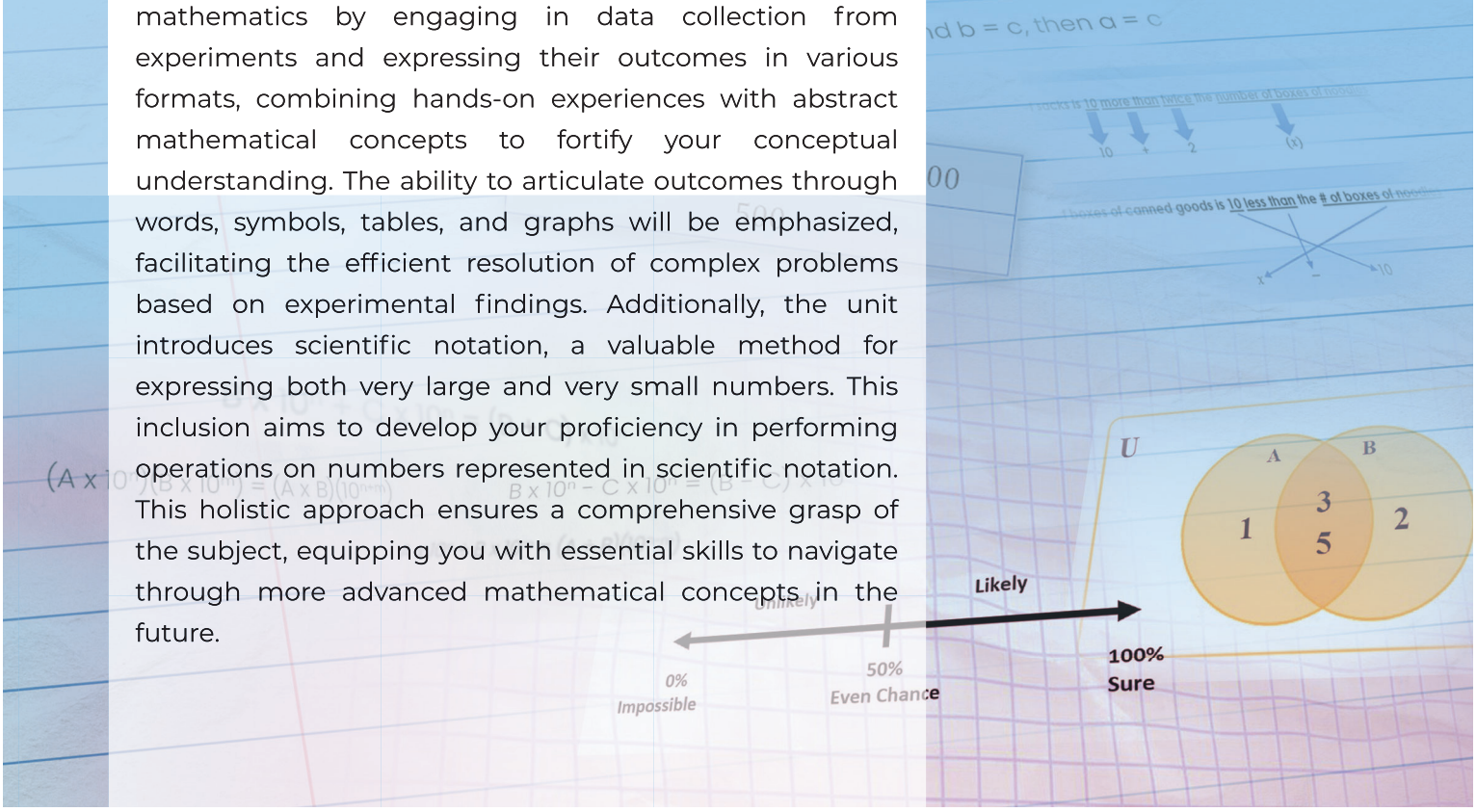
Rubric

Criteria	Excellent (4)	Proficient (3)	Developing (2)	Beginning (1)
Data Collection & Organization	Comprehensive and well-executed data collection; data neatly organized in tables.	Good data collection; mostly neat data organization.	Basic data collection; some errors in data organization.	Incomplete or incorrect data collection; poor data organization.
Graph Construction & Interpretation	Accurate and clear graphs; insightful interpretation of data.	Mostly accurate graphs; good interpretation of data.	Some inaccuracies in graphs; basic interpretation.	Incorrect or unclear graphs; poor or no interpretation.
Integer Understanding & Application	Deep understanding of integers; creative and accurate real-life applications.	Good understanding; mostly accurate applications.	Basic understanding; some inaccuracies in applications.	Poor understanding; incorrect or no real-life applications.
Integer Operations & Absolute Value	Correct and well-explained operations; clear understanding of absolute value.	Mostly correct operations; good explanation and understanding of absolute value.	Some errors in operations; basic understanding of absolute value.	Incorrect operations; poor or no understanding of absolute value.
Presentation & Clarity	Engaging, clear presentation with thorough explanations and confident responses.	Good presentation; clear explanations and good responses.	Adequate presentation; some unclear explanations and hesitant responses.	Poor presentation; unclear explanations and weak responses.

UNIT 4

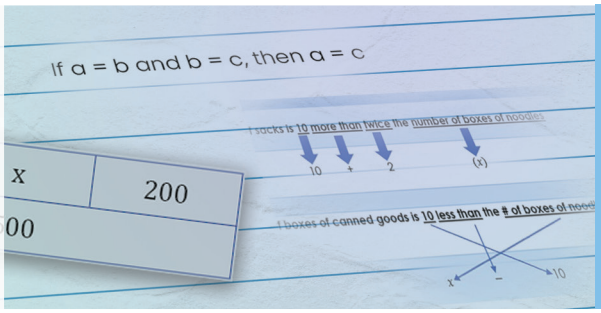
Your journey into the captivating world of mathematics and data representation begins in this unit. You will start by mastering the art of solving straightforward equations portrayed through bar models, effectively identifying unknowns. Simultaneously, you will acquire a solid understanding of distinguishing between variables and constants within algebraic expressions, enhancing your foundational skills. As you progress, you will evaluate algebraic expressions by substituting given values for variables and translating verbal descriptions into algebraic forms, refining your analytical and problem-solving abilities. The focus will extend to elucidating the properties of equality and manipulating algebraic equations to solve for one variable concerning others. This knowledge will equip you to inscribe equations in algebraic formats and derive values for non-negative unknowns, empowering you to tackle a myriad of problems involving algebraic expressions and equations.

Furthermore, you will gain practical insights into mathematics by engaging in data collection from experiments and expressing their outcomes in various formats, combining hands-on experiences with abstract mathematical concepts to fortify your conceptual understanding. The ability to articulate outcomes through words, symbols, tables, and graphs will be emphasized, facilitating the efficient resolution of complex problems based on experimental findings. Additionally, the unit introduces scientific notation, a valuable method for expressing both very large and very small numbers. This inclusion aims to develop your proficiency in performing operations on numbers represented in scientific notation. This holistic approach ensures a comprehensive grasp of the subject, equipping you with essential skills to navigate through more advanced mathematical concepts in the future.



LESSON 1

Introduction to Algebra



Objectives

- Solve simple equations represented by bar models to find unknowns.
- Distinguish a variable from a constant in an algebraic expression.

Examine

This is your first lesson on algebra. It has many real-life applications. Determining the total cost of items purchased, considering discounts or taxes, involves the use of algebraic expressions. Creating budgets and planning expenses involve algebraic expressions. Adjusting ingredient quantities in recipes based on the number of servings requires using algebraic expressions. Keeping track of scores or calculating statistics in sports involves algebra. Calculating travel time, distance, or speed involves algebraic formulas. Which of these fields interest you?

In this lesson, we are to learn to solve simple equations using bar models to uncover unknown values and gain the ability to differentiate between variables, representing unknowns, and constants, denoting fixed values, within algebraic expressions.

Algebra

- Introduction to Algebra
- Algebraic Expressions
- Algebraic Equations
- Problem Solving Algebra

Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).



Recall

Encircle the letter of the correct answer.

1. What is the solution for "a" in the equation 4a + 8 = 24?

a. 2 b. 4 c. 6 d. 8

2. What is the variable in the expression 3x + 2?

a. 3
b. x
c. 2
d. None of the above

3. Which of the following is a constant in the expression 2a + 3b - 6?

a. 2 b. a c. 3b d. -6

4. When solving an equation like 2a + 4 = 12, what does "a" represent in the problem?

a. The solution
b. The constant
c. The variable
d. The answer

5. What is the significance of the "+" symbol in the equation 5p + 3 = 18?

a. It represents subtraction
b. It separates variables and constants
c. It signifies multiplication
d. It indicates addition

6. Which part of the expression 2a - 5 represents a variable?

a. 2a
b. -
c. 5
d. None of the above

7. If the bar model representation shows 5x - 3 = 12, what is the value of "x"?

a. 3 b. 4 c. 5 d. 6

8. Apply your understanding to identify the variable in the expression 4x + 3.

a. 4
b. x
c. 3
d. None of the above

9. Sarah has some marbles. If she adds 5 more marbles to her collection, she will have a total of 12 marbles. How many marbles does Sarah currently have?

a. 5 b. 7 c. 12 d. 17

10. Emily has some stickers. She gave away 3 stickers to her friend, and now she has 9 stickers left. How many stickers did Emily have initially?

a. 3 b. 6 c. 9 d. 12



Explore

Before we start discussing the lesson, let us recall the concept of variables.

A variable is a letter or symbol that represents an unknown number.

Here are some examples of variables.

x = the number of apples Kevin has
y = the number of apples and oranges Kevin has
n = the number of bags of chips

Let us solve a problem involving a variable. Your class is going on a trip to the National Museum. The trip costs PhP 500 for each student. Included in that price is PhP 200 for the entrance fee and the cost of 2 transportation fees back and forth. Each of the transportation fees cost the same price.

Write an equation the cost of the trip and determine the price of the transportation fees.

Let us visualize the lesson using the bar model. In mathematics, visual models often assist in understanding abstract concepts, and the bar model is one such tool. It's primarily used in representing and solving equations, especially in earlier stages of algebraic understanding.

x	x	200
500		

where x = the cost of transportation fee going to the museum

Here is the equation based on the bar model.

$$x + x + 200 = 500$$

We add the x's.

$$2x + 200 = 500$$

To solve this problem, take away the PhP 200 for the entrance fee.

We remove 200 from both sides of the equation. This is the new bar model.

x	x
300	

$$2x = 300$$

Cut in half the PhP 300 to get the value of x.

$$300 \div 2 = x$$
$$x = 150$$

Therefore, x = 150. Moreover, the transportation fee going to the museum costs PhP 150. This is also the cost of going back from the museum.

In algebra, expressions and equations often contain both variables and constants. Understanding the difference between these two is crucial.

A variable is a symbol (usually a letter) that represents an unknown or changing value in an expression or equation.

A constant is a specific value in an expression that does not change. In algebraic expressions, constants can be numbers that stand alone.

Here are some examples.

In the expression 2x + 3, "x" is the variable because it represents an unknown value that can change. "2" is the coefficient of the variable.

In the same expression, "3" is a constant because it is a fixed value that does not change.

In simple terms, while variables can represent many different values and can change depending on the situation, constants always represent the same value. Recognizing this difference is foundational in algebra, as it aids in simplifying expressions, solving equations, and understanding the relationships between different mathematical terms.



Try This

Answer the following on your own.

1. Solve 2x + 3 = 11 by finding the value of the unknown variable. Draw a bar model to represent this equation.
2. Identify all the variable/s and constant/s in the expression, 4y + 2b + 7.



Key Points

- Solve straightforward equations using visual aids called bar models, to easily find unknown values efficiently.
- Algebraic expressions and equations are composed of variables (represented by letters and symbolizing changing values) and constants (fixed numerical values).



Do

I. Encircle the letter of the correct answer.

1. In the equation 2x + 5 = 15, what is the value of "x"?

a. 5 b. 10 c. 15 d. 20

2. If a bar model represents the equation 3y + 6 = 12, what is the value of "y"?

a. 2 b. 3 c. 4 d. 6

3. Identify the constant in the expression 4y - 7.

a. 4
b. y
c. -7
d. None of the above

4. If you see "x" in an algebraic expression, what does it most likely represent?

a. A constant
b. An unknown or changing value
c. An operation
d. A variable

5. What is the role of constants in algebraic expressions?

a. To confuse students
b. To represent changing values
c. To add complexity
d. To represent fixed values

6. If a bar model represents the equation 4y - 8 = 24, what does the "4y" part of the equation signify in the context of the problem?

a. The unknown value
b. The constant
c. The total value
d. The variable

7. Using a bar model, find the solution for "y" in the equation 2y + 7 = 15.

a. 4 b. 5 c. 6 d. 7

8. In the expression 5y - 9, apply your knowledge to identify the constant.

a. 5
b. y
c. -9
d. None of the above

9. Mark is organizing a bake sale. He baked 18 cookies, and he wants to divide them equally into 6 bags. How many cookies will be in each bag?

a. 2 b. 3 c. 6 d. 9

10. There are 24 students in a class, and they need to be divided into 4 equal groups for a project. How many students will be in each group?

a. 4 b. 6 c. 8 d. 12

II. Give what is asked.

1. Given the expression: 5x + 7

   Question: Which part represents a variable?

2. Given the expression: 3y - 4

   Question: Which part represents a constant?

3. Given the expression: 7z + 9

   Question: What symbol stands for a variable?

4. Given the expression: 8 - w

   Question: Which part of the expression does not change its value?

5. Given the expression: 6a + 5

   Question: Which term is a combination of a constant and a variable?

6. Given the expression: b + 12

   Question: If the value of the variable changes, which part of the expression would be affected?

7. Given the expression: 9 - 2c

   Question: Which term represents a constant?

8. Given the expression: 3d + 7

   Question: Which term has a letter and number combined?

9. Given the expression: 4 + e

   Question: If e stands for eggs and you don't know how many eggs there are, which part of the expression is the variable?

10. Given the expression: 7f - 6

    Question: Which term remains the same regardless of the value of the letter in the expression?

III. Draw the bar models and solve the following problems.

1. Jenny has twice the amount of money as Mark. If we represent Mark's money with a bar, Jenny's money can be represented with two bars. Together, they have 450 pesos. How much money does Mark have?
2. A bookshop sold 3 similar books for a total of 600 pesos. If each book costs the same, how much is one book?

3. Anna saved up 4 times as much as Bryan. Together, they have 500 pesos. How much did Anna save?
4. A chocolate bar is divided into 5 equal parts. If 3 parts weigh 120 grams, how much does the entire chocolate bar weigh?
5. Karla's age is 3 times that of her younger brother. In total, their ages add up to 48 years. How old is Karla's younger brother?



Brainstorm

Your teacher will find you a partner. You will work in pairs to distinguish a variable from a constant in algebraic expressions. Each person will be creating three (3) algebraic expressions. They will exchange and analyze the expressions and identify the variables and constants within them. The pair will record their findings and reasoning in their notebooks. After analyzing several expressions, pairs will discuss their results and reasoning with each other.



Design

You will work together to solve simple equations represented by bar models. Your teacher will divide the class into small groups of 4 members each and provide each with a problem. You will first create the bar models, guess the corresponding equations, and then collaborate to solve them. Discussions within groups to explore various problem-solving strategies are encouraged during the activity. Each group will present their solutions to the class, explaining their thought processes and how they used the bar models to find unknown values.

Group 1: In a game, Mia scored some points in the first round and doubled her score in the second round. If her total score is 36, and you represent her points in the bar model as $x + 2x = 36$, what is Mia's score in the first round?

Group 2: Emma and Liam have some money together. Emma has PhP 10 more than Liam. If they represent their money in a bar model as $x + (x + 10) = 50$, how much money does Emma have?

Group 3: A farmer sold some bags of apples in the morning and then sold twice as many bags in the afternoon. If he sold a total of 30 bags, and you represent the bags sold in the bar model as $x + 2x = 30$, how many bags did he sell in the afternoon?

Group 4: A baker is making pies for a school event. She baked some apple pies and twice as many cherry pies. If she baked a total of 24 pies, and you represent the pies in the bar model as $x + 2x = 24$, how many cherry pies did she bake?

Group 5: A toy store sold some red cars and green cars. The number of red cars sold was 4 more than the green cars. If they represent the cars in the bar model as $x + (x + 4) = 30$, how many green cars were sold?

Group 6: Jenny saved some money from her allowance, and then her parents gave her PhP 15 as a gift. If she now has a total of PhP 45 and you represent her savings in the bar model as $x + 15 = 45$, how much money did Jenny save initially?

Group 7: David's age is currently three times that of his younger brother. If you represent his age using a bar model as $3x = 21$, what is the age of his younger brother?

Group 8: A classroom has some desks in the first row, and there are twice as many desks in the second row as in the first row. If there are a total of 24 desks, and you represent the desks in the bar model as $x + 2x = 24$, how many desks are in the second row?

Group 9: Sarah buys pens for herself and her friend. She would have twice as many pens as her friend. There are 36 pens in total. How many pens does her friend have?

Group 10: A store sells some computers in the morning and then sells twice as many computers in the afternoon. If they sold a total of 45 computers, and you represent the computers in the bar model as $x + 2x = 45$, how many computers were sold in the morning?

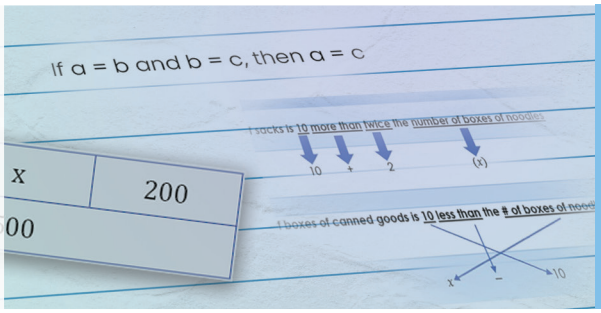


Fun Facts

Algebra originates from the Arabic word "al-jabr," meaning "reunion of broken parts." The system of algebra as we know it today traces its roots back to ancient civilizations like Babylonians and Egyptians.

LESSON 2

Algebraic Expressions





Objectives

- Simplify algebraic expressions.
- Evaluate algebraic expressions given the value/s of the variable/s.
- Translate verbal phrases into algebraic expressions.



Examine

Algebraic expressions have many practical applications. Here are some of them.

Imagine calculating the total cost of buying multiple items at a store. You can use algebraic expressions to represent the cost of each item and find the total expenditure, incorporating discounts or taxes. Have you done this before? What items did you want to buy?

Creating a budget involves allocating amounts for different expenses like groceries, bills, entertainment, and savings. Algebraic expressions can help in managing budgets and setting financial goals. Do you think algebra can help you in your budget?

Algebra

- Introduction to Algebra
- Algebraic Expressions
- Algebraic Equations
- Problem Solving Algebra



Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).

When planning a journey or commute, algebraic expressions can be used to calculate distance, speed, and time. Using algebra, one can manipulate the formula to find distance or time based on the given information.

Calculating the area of a garden or land plot involves using algebraic expressions. For instance, determining the area of irregularly shaped gardens using formulas for different shapes.

In cooking, adjusting ingredient quantities when scaling a recipe up or down can be represented using algebraic expressions. If a recipe serves four people and you need to serve eight, you'll need to double the ingredients, which can be expressed algebraically. Where else can algebra help us?

In this lesson, we are to learn to evaluate algebraic expressions when provided with variable values and acquire the skill to convert verbal descriptions into algebraic expressions, enhancing our proficiency in both solving and formulating mathematical equations.



Recall

Encircle the letter of the correct answer.

1. What does the expression $4y - 7$ represent if $y = 3$?

a. 1 b. 13 c. 12 d. 5

2. Translate the phrase "the difference between a number and 9" into an algebraic expression.

a. $9x$ b. $x - 9$ c. $x + 9$ d. $x \div 9$

3. What is the value of $3(x - 2)$ when $x = 7$?

a. 21 b. 9 c. 15 d. 18

4. If $p = 8$ and $q = 3$, what is the value of $p + q$?

a. 11 b. 24 c. 5 d. 15

5. What does the expression $2y - 5$ represent if $y = 4$?

a. 3 b. 13 c. 7 d. 10

6. Translate the phrase "the sum of a number and 10" into an algebraic expression.

a. $x + 10$ b. $10x$ c. $10 - x$ d. $x \div 10$

7. What is the result of evaluating $3(x + 2)$ when $x = 4$?

a. 12 b. 24 c. 18 d. 30

8. Evaluate $2(3x - 1)$ when $x = 5$.

a. 28 b. 14 c. 26 d. 8

9. Sarah has a certain number of marbles. If she adds 5 marbles to her collection, the algebraic expression for the total number of marbles she has is:

a. $5 - x$ b. $x + 5$ c. $5x$ d. $5 \div x$

10. John is twice as old as his younger brother. If his brother's age is represented by the variable y, what is the algebraic expression for John's age?

a. $y - 2$ b. $2y$ c. $y \div 2$ d. $y + 2$



Explore

The outreach club in your school is scheduling the repacking of goods for the victims of the earthquake on a nearby island next week. Before they can decide on the content of the food packs, they first need to determine the actual number of donations from the different sponsors. The club's outreach head noted the following donations namely, sacks of rice, boxes of noodles, and boxes of canned goods. Also, the number of sacks is 10 more than twice the number of boxes of noodles. The number of boxes of canned goods is 10 less than the number of boxes of noodles.

If x represents the number of boxes of noodles, then:

a. How many sacks of rice are there?
b. How many boxes of canned goods are there?

Representation: Let x be the number of boxes of noodles.

a	
Relationship	number of sacks is 10 more than twice the number of boxes of noodles
Algebraic Expression	10 + 2 (x)
b	
Relationship	number of boxes of canned goods is 10 less than the # of boxes of noodles
Algebraic Expression	x − 10

Therefore, there are 10 + 2x sacks of rice and x - 10 boxes of canned goods.

What is the total number of donations in terms of x?

Sacks of rice	+	Boxes of noodles	+	Boxes of canned goods
10 + 2x	+	x	+	x - 10

$$10 + 2x + x + x - 10$$
$$(2x + x + x) + (10 - 10)$$
$$(x + x + x + x) + 0$$
$$4x$$

Therefore, there are a total of 4x donations.

In the solution, you must have observed that the terms were grouped as constants and another group of algebraic expressions with variable x. Also, when 2x, x, and another x are added (2x + x + x), the sum is equal to x + x + x + x or equal to 4x. The coefficient 4 is the sum of the coefficients (2 + 1 + 1)x. The addition of the coefficients of the three is possible because they are considered similar terms. In the problem, the variable x was replaced with a constant value, and the given expression was evaluated resulting in a constant value. This shall be discussed thoroughly in the evaluation of algebraic expressions.

In algebra, similar or like terms are terms that have the same variables and the same exponents in these variables. Otherwise, they are called not similar or unlike terms. In the problem about donations, the same variable of x was used and the same exponent of 1 was used. Like terms were seen.

Another example of like terms is 3y and 12y. Their same variable is y.

For unlike terms, an example is 3x and 5y. The variables are x and y. Another example is 8x and $9x^3$. The variables are x and x^3. The same variable is used but the exponents are different.

In algebra, addition or subtraction of algebraic expressions can be easily performed if you remember that:

In adding or subtracting algebraic expressions, you can only combine like terms into one single expression.

In evaluating algebraic expressions, substitute specific values assigned to the variables involved in the expression and simplify it. Here is an example.

Let us evaluate 7x + 2y when x = 5 and y is -3.

When x = 5 and y = -3, and 7x + 2y, then:

$$7(5) + 2(-3)$$
$$35 + (-6)$$
$$29$$

Let show what happens when we evaluation the same algebraic expression and use the values of x = -4 and y = 2.

$$7(-4) + 2(2)$$
$$-28 + 4$$
$$-24$$

Aside from simplifying and evaluating algebraic expressions, we can also translate verbal phrases into algebraic expressions. These are some of the common words and phrases that are used in algebraic expressions, such as:

"a number" (can be represented as 'x' or any variable)
"more than" (can be represented as '+')
"less than" (can be represented as '−')
"twice" (can be represented as '2x')
"the sum of" (can be represented as '+')
"the difference between" (can be represented as '−')
"product of" (can be represented as '×')
"quotient of" (can be represented as '÷')

Here is an example. Translate the phrase "A number increased by 5" in algebraic expression. Since there is an increase, we use the + sign. x + 5 is the algebraic expression.



Try This

Translate the following verbal phrases into algebraic expressions.

1. A number increased by 8
2. Three times the difference between a number and 7
3. The product of 6 and a number
4. The quotient of a number and 3
5. The product of two consecutive even numbers



Key Points

- Evaluating algebraic expressions involves substituting specific numbers for variables in algebraic expressions and then solving the expression to find the value.
- You need to understand how to calculate with variables and apply these skills to solve problems.
- Translating verbal phrases into algebraic expressions is learning to convert everyday language into mathematical language.
- You need to identify key words and phrases in sentences and use them to write algebraic expressions.
- This skill is crucial for solving word problems and representing real-world situations with algebra.



Do

I. Encircle the letter of the correct answer.

1. What is the algebraic expression for "the sum of a number and 8"?

a. $x + 8$ b. $8x$ c. $x - 8$ d. $x \div 8$

2. If $x = 5$, what is the value of $2x$?

a. 10 b. 7 c. 25 d. 2

3. Translate the phrase "three times a number" into an algebraic expression.

a. $3 + x$ b. $3x$ c. $x - 3$ d. $x \div 3$

4. Which algebraic expression represents "the difference between a number and 5"?

a. $x + 5$ b. $x - 5$ c. $5x$ d. $x \div 5$

5. What is the result of evaluating $3x + 2$ when $x = 4$?

a. 12 b. 10 c. 14 d. 22

6. Translate the phrase "the product of 6 and a number" into an algebraic expression.

a. $6 + x$ b. $6 - x$ c. $6x$ d. $x \div 6$

7. If $x = 5$ and $y = 3$, what is the value of $2x - 3y$?

a. 6 b. 11 c. 3 d. 1

8. Translate the phrase "the difference between a number and 9" into an algebraic expression.

   a. x - 9 b. 9x c. x + 9 d. x ÷ 9

9. Suppose you have a rectangular garden with a length of x meters and a width of 4 meters. What is the algebraic expression for the area of the garden?

   a. 4x b. x - 4 c. x ÷ 4 d. x + 4

10. Given two expressions a + 2b and 3a - b. If a = 2 and b = 3, which expression has a higher value?

    a. a + 2b
    b. 3a - b
    c. Both are equal
    d. Cannot be determined

II. Write the following algebraic expressions in symbols.

1. A number x decreased by 10
2. The product of a and b
3. 8 more than a number n
4. The sum of 5a and 3b
5. The difference between the product of a and b, and the quotient of a and b
6. The sum of $4a^2$ and 11y
7. Multiply 11p by 8q
8. The difference between 15k and 10l
9. 7 is taken away from the product of 3x and 5y
10. The sum of 15xy and square root of z is subtracted from the quotient of 2x and z

III. Determine whether the given pairs of algebraic expressions are like terms or not. Write L if they are similar and N if otherwise.

1. 2x and 2xy
2. $5x^2$ and $-5x^2$
3. $4x^3y^2$ and $-9y^2x^3$

IV. Given that x = 5 and y = -2, determine the value of the following.

1. 4x + 9y
2. $2xy - 4y^2$
3. $7x^3y^2 - 2xy^3$

V. Simplify the algebraic expressions.

1. 3x + 5x + 7
2. 4y + 2y - y
3. 7a + 3b - 4a - b
4. 5m + 4n + m - 2n
5. 3(p + 4) - 2(p - 3)



Brainstorm

Your teacher will help you find a partner. You will work in pairs to evaluate algebraic expressions given the values of the variables. Each person will create three (3) algebraic expressions and set the given values of the variables. You will exchange with your partner and evaluate the expressions given the values of the variables. Record your findings and reasoning in your notebook. After analyzing several expressions, discuss the results and reasoning with each other.



Design

Your teacher will divide the class into groups with 5 students. Each student will be given expressions to analyze and answer the questions that follow. Each student is to present their work afterward to their groupmates.

Student 1: $5x^2$ and $-5x^2$

Student 2: $6xy^2$ and $\frac{1}{2}xy^2$

Student 3: $\frac{2}{3}y^3$ and $\sqrt{2y^3}$

Student 4: a^2b^3 and $-14a^2b^3$

Student 5: $22jl^2k^3$ and $\frac{jl^2k^3}{8}$

1. What are the variables in the given pairs of algebraic expressions?
2. Do they have the same exponents?
3. Are they like terms or not?

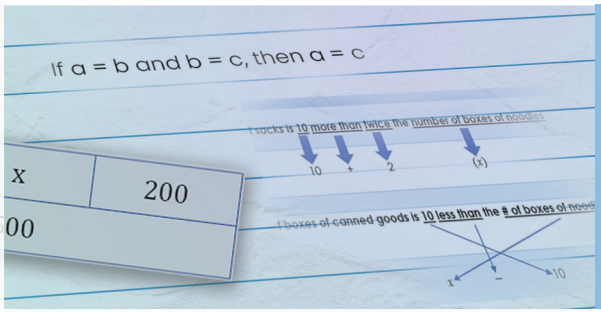


Fun Facts

Al-Khwarizmi, the Father of Algebra: Muhammad ibn Musa al-Khwarizmi, a Persian mathematician, wrote a book called "Al-Kitab al-Mukhtasar fi Hisab al-Jabr wal-Muqabala," which served as a fundamental guide in introducing algebraic methods and concepts.

LESSON 3

Algebraic Equations



Objectives

- Illustrate the properties of equality.
- Solve one variable in terms of the other variables in a formula.
- Write equations in algebraic form.

Examine

Algebraic equations have practical utility outside the classroom. Budget planning, travel time, temperature conversion, sports activities, and growth of plants are some real-life situations involving algebraic equations. Where else can we use algebraic equations?

In this lesson, we are to explore the fundamentals of algebraic equations and expressions, focusing on illustrating the properties of equality, solving for one variable within formulas, and effectively writing equations in algebraic form, providing a strong foundation for algebraic problem-solving and mathematical modeling.

Algebra

- Introduction to Algebra
- Algebraic Expressions
- Algebraic Equations
- Problem Solving Algebra

Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).



Recall

Encircle the letter of the correct answer.

1. Which mathematical concept involves expressing one variable in terms of other variables in a formula?

a. Addition b. Subtraction c. Multiplication d. Substitution

2. Which of the following represents writing equations in algebraic form?

a. $3 \times 4 = 12$ b. $2 + x = 7$ c. $5 + 2 = 7$ d. $2 \div 4 = 0.5$

3. What is the primary purpose of the properties of equality in mathematics?

a. Solving complex equations
b. Simplifying algebraic equations
c. Memorizing mathematical rules
d. Describing geometric shapes

4. Which mathematical property allows you to perform the same operation on both sides of an equation without changing the equation's solution?

a. Reflexive property
b. Symmetric property
c. Addition property
d. Transitive property

5. If you want to solve for x in the equation $3x + 5 = 11$, which step would you take first?

a. Subtract 5 from both sides
b. Divide both sides by 3
c. Add 11 to both sides
d. Multiply both sides by 3

6. Which of the following equations represents the symmetric property in action?

a. $2x = 2x$ b. $4 + y = y + 4$ c. $2x = 5(2)$ d. $5 - 2 = x - 2$

7. Which of the following equations demonstrates the application of the multiplication property?

a. $3 + y = 6 + 3$ b. $3y = 6(3)$ c. $5z = 5z$ d. $a + 10 = 10 + a$

8. If you need to find the value of y in the equation $2y + 7 = 15$, which step would you take after isolating 2y on one side of the equation?

a. Subtract 7 from both sides
b. Divide both sides by 2
c. Add 15 to both sides
d. Multiply both sides by 2

9. Which equation correctly translates the statement: "Triple a number subtracted by 4 is equal to 20"?

a. $3x + 4 = 20$ b. $x - 4 = 20$ c. $3x - 4 = 20$ d. $x \div 3 - 4 = 20$

10. The area, A of a rectangle is given by the formula A = lw where l is the length and w is the width. If you're given that A = 40 and w = 8, which equation would you use to solve for l?

a. l = 40 + 8 b. l = 40 - 8 c. $l = \frac{40}{8}$ d. l = 40(8)



Explore

In the previous lesson, we translated words into mathematical expressions. We can do the same for equations.

Imagine you're given a word problem or a real-life situation, and you need to express it mathematically. This is where writing equations in algebraic form comes into play.

An equation is a mathematical statement that asserts the equality of two expressions, meaning that they represent the same quantity. For instance, if I say, "The sum of two numbers is fifteen," we can represent it with an equation like: x + y = 15. Remember that we have numbers, operations, and variables.

You have learned how to solve simple equations in Lesson 22. We did not state it but we already used the properties of equality.

Now, here are some properties of equality which would be good to keep in mind. These would greatly help us to solve equations.

Reflexive Property:

A number is equal to itself.

a = a

If p = 5, then by the reflexive property, p = p so 5 = 5.

Symmetric Property:

If one quantity equals another, then the sides can be reversed.

If a = b then b = a

If q = 7, then by the symmetric property, 7 = q.

Transitive Property:

If one quantity equals a second, and the second quantity equals a third, then the first equals the third.

If a = b and b = c, then a = c

If x + y = 8, and 8 = z, then x + y = z.

Addition and Subtraction Properties:

If you add (or subtract) the same number to both sides of an equation, the two sides remain equal.

If $a = b$, then $a + c = b + c$ and $a - c = b - c$

If $r = 6$, then $r + 3 = 6 + 3$.

For subtraction, if $s = 4$, then $s - 9 = 4 - 9$.

Multiplication and Division Properties:

If you multiply (or divide) both sides of an equation by the same nonzero number, the two sides remain equal.

If $a = b$, then $ac = bc$ and $a \div c = b \div c$ provided $c \neq 0$

If $t = 10$, then $5t = 10(5)$.

For division, if $u = 6$, then $u \div 3 = 6 \div 3$.

These properties are essential to solving equations.

From our previous lesson on evaluating algebraic expressions, we know how to plug-in the values of the variables and simplify.

For this part, we are going to express a variable in terms of other variables. We do not need to solve and find an exact value. We use the properties of equality to isolate a variable.

In some situations, especially in advanced mathematics and science, it's useful to express one variable as a combination of other variables, even if we don't know the exact values of any of them.

Here is an example.

We are given the equation $d = t + a$. We want to express the variable t in terms of the other variables d and a.

Let us use the properties of equality. Use the subtraction property of equality to subtract a from both sides of the equation.

$$d - a = t + a - a$$
$$d - a = t$$

Use the symmetric property of equality to switch the positions.

$$t = d - a$$

Here is another example.

The area of a rectangle is given by the formula, $A = lw$. We want to express the variable l in terms of the other variables A and w.

Let us use the properties of equality. Use the division property of equality to divide w to both sides of the equation.

$$\frac{A}{w} = \frac{lw}{w}$$

$$\frac{A}{w} = l$$

Use the symmetric property of equality to switch the positions.

$$l = \frac{A}{w}$$

Here are more questions on the properties of equality.



Try This

Answer the following questions individually.

1. If you have the equation 3x + 5 = 11, and you subtract 5 from both sides to isolate 3x, which property of equality are you using?
2. In the equation 4a = 20, if you divide both sides by 4 to solve for a, which property of equality is applied?
3. If you rearrange the equation a + b = b + a, which property of equality explains this reordering?



Key Points

- There are different properties of equality- reflexive, symmetric, transitive, addition, subtraction, multiplication, and division.
- You can translate verbal descriptions or word problems into algebraic equations using variables, constants, and mathematical operations.
- You can solve equations with one variable in terms of other variables by isolating the variable by performing operations on both sides of an equation, using principles such as addition, subtraction, multiplication, and division.



Do

I. Encircle the letter of the correct answer.

1. In algebra, which property allows you to add the same value to both sides of an equation without changing its solutions?

a. Reflexive property
b. Symmetric property
c. Transitive property
d. Addition property

2. Which of the following equations is written in algebraic form?

a. 3 + 5 = 8 b. 2(4) = 16 c. x - 7 = 3 d. 9 ÷ 3 = 3

3. What is the first step in solving an equation involving one variable?

a. Multiply both sides by the variable
b. Add or subtract numbers from both sides
c. Divide both sides by a constant
d. Simplify the equation

4. Which of the following represents the concept of writing equations in algebraic form?

a. 4 + 2 = 6 b. 3x = 12 c. 2 - 7 = -5 d. 9 ÷ 3 = 3

5. In an algebraic equation, what is the role of variables?

a. To provide a specific numerical solution
b. To represent unknown quantities
c. To perform mathematical operations
d. To illustrate geometric shapes

6. Which of the following equations is written in algebraic form?

a. 3 + 5 = 8
b. 2(1 + 3) = 2(3) + 6
c. 7 - x = 2
d. 4(4) = 16

7. In the equation 5x - 3 = 12, what is the value of x after solving for it?

a. 2 b. 3 c. 15 d. 5

8. When you have the equation 3(x - 2) = 15, what should you do first to solve for x?

a. Add 2 to both sides
b. Subtract 3 from both sides
c. Divide both sides by 3
d. Multiply both sides by 3

9. Juanita wants to solve for y in terms of x using the formula xy = 12. Which expression correctly represents y in terms of x?

a. y = 12 - x b. $y = \frac{12}{x}$ c. y = 12x d. y = x + 12

10. A rectangular garden has a length that is 3 m more than its width. Which of the following represents the length in algebraic form if w stands for the width.

a. w + 3 b. w - 3 c. 3w d. w ÷ 3

II. Write EX for algebraic expressions and EQ for algebraic equations.

1. abc

2. a = b

3. a + b = b + c

4. c + 5 = 10

5. 3x = 9

6. 4x + 7

7. 3x + 5x - 5

8. y + 4 = 0

9. xy

10. 3 - z

III. Write an algebraic equation for each.

1. The sum of 3, 4, and n is 10.

2. Twice y is 20.

3. The variable x decreased by 6 is 5.

4. The variable m multiplied by 8 gives 64.

5. The product of 5, 10, and x is 500.

6. Half of x is 3.5.

7. 8 more than the product of 9 and 7 is n.

8. 5 less than 14 is equal to n.

9. One-third as large as m is 20.

10. The sum of 5 and 6 increased by the sum of 9 and 10 is x.

IV. Solve one variable in terms of the other variables in a formula.

1. Given A = lw where A is the area, l is the length, and w is the width, solve for w in terms of A and l.

2. Given P = 2l + 2w where P is the perimeter of the rectangle, solve for l in terms of P and w.

3. The speed of an object is given by the formula $v = \frac{d}{t}$ where v is the speed, d is the distance, and t is the time. Solve for t is terms of v and d.

4. Using the formula for an area of a triangle, $A = \frac{1}{2}bh$ where b is the base and h is the height, solve for h in terms of A and b.



Brainstorm

Your teacher will pair you up with a partner. Assign who will be Person A and Person B. Each pair will have to solve one variable in terms of the other variables in a formula using the properties of equality. You will discuss your work with your partner afterward.

Person A

In the formula $C = 2\pi r$, where C represents the circumference and r represents the radius of a circle, isolate the variable r in terms of the variable C and the constants 2 and π.

Person B

You are using the formula $I = PRT$ to calculate the interest (I) on a loan. Isolate the variable P (principal) in terms of the variables R (interest rate) and T (time).



Design

The teacher will divide the class into groups of 4 members each. Assign yourselves to be Person A, Person B, Person C, and Person D. Each of you will be given a set of equations and you will identify the properties of equality illustrated. Once all of you are done, explain to your classmates your work.

Person A

1. $11 = 11$
2. $r = 1$ and $1 = r$
3. $x = 5$ and $5 = y$ then $x = y$
4. $z - 15 = 6 - 15$
5. $a \div 2 = 4 \div 2$

Person B

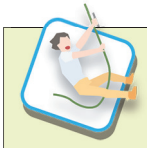
1. $5b = 20(5)$
2. $s = s$
3. $m = 2$ and $2 = m$
4. $c = 50$ and $50 = j$ then $c = j$
5. $k + 9 = 16 + 9$

Person C

1. i - 2 = 23 - 2
2. p ÷ 6 = 24 ÷ 6
3. 3 = 3
4. a = 3 and 3 = a
5. l = 15 and 15 = h then l = h

Person D

1. m = 2 and 2 = z then m = z
2. 5 + n = 5 + 4
3. 8o = 9(8)
4. j = j
5. 4 = o and o = 4

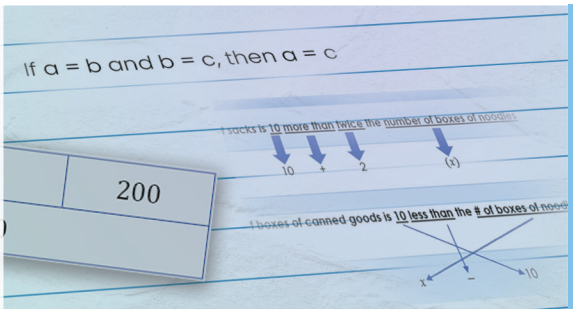


Fun Facts

The Babylonians and ancient Egyptians used basic forms of algebraic equations to solve problems related to trade, taxation, and construction thousands of years ago.

LESSON 4

Problem Solving Algebra





Objectives

- Find the value of an unknown in an equation where the unknown is non-negative.
- Solve problems involving algebraic expressions and formulas.



Examine

In the past 3 lessons, we discovered the many real-life situations involving algebra. We continue our discussion here. We try to answer the essential question, "How can algebraic expressions and equations be used to solve real-life problems involving unknown quantities?"

In this lesson, we are to learn to determine the value of an unknown in non-negative equations and apply algebraic expressions and formulas to solve various mathematical problems, enhancing our problem-solving abilities and understanding of algebraic concepts.

Algebra

- Introduction to Algebra
- Algebraic Expressions
- Algebraic Equations
- Problem Solving Algebra



Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).



Recall

Here are several questions we keep in mind to solve problems involving algebra.

What strategies can be employed to translate word problems into algebraic equations efficiently?

How does identifying and applying the correct operations in an equation assist in finding the solution?

Why is it crucial to verify the solutions obtained by substituting them back into the original problem?



Explore

Algebra revolves around the process of discovering unknown values or solutions you seek by utilizing known information. These unknowns could encompass variables such as the cost of a car loan, the quantity of soda required, or the height at which a water balloon can be thrown. Anything that remains uncertain or undisclosed falls under the category of unknowns.

The various techniques and principles you encounter in algebra are merely tools employed to manipulate equations and facilitate the quest for these missing pieces of information. There exist guidelines governing actions like multiplication and rearrangement of terms across an equal sign, but ultimately, they serve as strategies and tactics designed to aid in the quest for the elusive missing data you are in pursuit of.

For example, you are about to end your algebra lessons for the year, and you scanned through your quizzes and seatwork and saw that you got the following ratings.

95, 89, 94, 90, 91, 93, 92

You decided to use your algebra skills and solve for your average algebraically. What is your average?

You know that to get your average, you first must get the sum of all your scores and divide it by the total number of activities you had. You also decided to use a variable, x, to represent your average.

Representation	x = average rating
Solution	$x = \frac{95 + 89 + 94 + 90 + 91 + 93 + 92}{7} = 92$
Answer	For the activities, your average rating is 92%.

Now that we've warmed up a bit, let us solve word problems involving equations.

To have a general guide, in solving word problems, let us follow these steps:

1. Identify your unknown and represent it using a variable.
2. Translate English sentences to mathematical equations.
3. Solve for the unknown variable using the properties we have learned.
4. After getting a value for the unknown variable, check your answer by substituting it back to your first equation.
5. Once you're sure of your answer, don't forget to always label it.

Let us apply these steps in solving a number relation problem.

The sum of two consecutive integers is 11. What are the two integers?

Representation	Let x be the first integer. Let x+1 be the second integer.
Translation	$x + x + 1 = 11$
Solution	$2x + 1 = 11$ $2x = 10$ $x = 5$
Checking	Substitute 5 to the first equation. $5 + 5 + 1 = 11$ $11 = 11$ Since $11 = 11$ is true, that means we got the correct answer.
Final Answer	If the first integer, x, is equal to 5, then $x + 1$ is 6. The first integer is 5 while the second integer is 6.

Let us also try solving a problem related to perimeters.

The perimeter of the rectangle is 16 meters. If its length is one less than twice the width, what is the measurement of both the length and the width?

Representation	We are asked to solve for the length and width of the rectangle. Let x be the width. Let 2x -1 be the length. Recall that to get the perimeter of a rectangle we just have to add its two lengths and two widths.
Translation	$x + x + 2x - 1 + 2x - 1 = 16$
Solution	$6x - 2 = 16$ $6x = 18$ $x = 3$
Checking	Substitute 3 to the first equation. $3 + 3 + 2(3) - 1 + 2(3) - 1 = 16$ $16 = 16$ Since $16 = 16$ is true, that means we got the correct answer!
Final Answer	Since $x = 3$, we now know that the width is 3 meters. On the other hand, the length is $2(3) - 1 = 5$ meters. The width and length of the rectangle are 3 meters and 5 meters, respectively.

Let us now try solving an age problem.

John is four years older than twice Mark's age. John's age today will be the same as Mark's age fourteen years from now. How old is John?

Representation	We are asked to solve for John's present age. Let x be Mark's age. Let $2x + 4$ be John's age. Also note that, 14 years from now, Mark's age will be $x + 14$. While John will be $2x + 4 + 14$ years old.
Translation	$2x + 4 = x + 14$
Solution	$2x + 4 = x + 14$ $2x - x = 14 - 4$ $x = 10$

Checking	Substitute 10 to the first equation. $2x + 4 = x + 14$ $2(10) + 4 = 10 + 14$ $24 = 24$ Since $24 = 24$ is true, that means we got the correct answer!
Final Answer	Since we know that $x = 10$ is Mark's present age, we can use this to get John's present age by substituting 10 to $2x + 4$. $2(10) + 4 = 24$ John is currently 24 years old.

Another common type of word problem you may encounter is a mixture problem.

A bag of classic candies from your favorite candy store costs PhP 50 per 100 grams, while a bag of premium candies costs PhP 100 per 100 grams. How much would 50 grams of classic candies and another 50 grams of premium candies cost you?

Representation	We are asked how much it would cost a mixture of classic and premium candies. Let x be the cost per gram of assorted candies. Before we start solving, let us first compute how much the cost is per gram of candy. Classic = PhP 50/ 100 grams = PhP 0.50/ gram Premium = PhP 100/ 100 grams = PhP 1/ gram
Translation	$0.50(50) + 1(50) = x(50 + 50)$
Solution	$25 + 50 = x(100)$ $75 = 100x$ $0.75 = x$
Checking	Substitute 0.75 to the first equation. $0.50(50) + 1(50) = 0.75(50 + 50)$ $25 + 50 = 0.75(100)$ $75 = 75$ Since $75 = 75$ is true, that means we got the correct answer!
Final Answer	Since we know that $x = 0.75$, our total cost is $0.75(50 + 50) = 75$. A mixture of 50 grams of classic candies and 50 grams of premium candies costs PhP 75.

Here is a problem related to discounts, mark-ups, and pricing.

There's an ongoing sale in your favorite retail store. The shirt that you've been wanting to buy originally costs PhP 500. If all items are subject to a 20% discount, how much will you pay for the shirt?

Representation	We are asked to solve for the discounted price of the shirt. Let x be the discounted shirt price.
Translation	$x = 500 - 500(0.2)$
Solution	$x = 500 - 100$ $x = 400$
Checking	$400 = 500 - 500(0.2)$ $400 = 500 - 100$ $400 = 400$ Since $400 = 400$ is true, that means we got the correct answer!
Final Answer	The discounted price of the shirt is PhP 400.

Lastly, let us solve an investment problem.

You have PhP 5000 in your bank account. If it has been there for two years and has earned at least PhP 50, what is the interest rate of the bank?

Representation	We are asked to solve the interest rate. Let x be the interest rate. Before we start, let us first remember that to solve for interest, we use the formula, Interest = Principal × Rate × Time
Translation	$50 = 5000(x)(2)$
Solution	$50 = 10\,000x$ $0.005 = x$
Checking	Substitute 0.005 to the first equation. $50 = 5000(0.005)(2)$ $50 = 50$ Since $50 = 50$ is true, that means we got the correct answer!
Final Answer	Transforming 0.005 to percent, we have 0.5%. The interest rate of the bank is at least 0.5%



Try This

Answer the following problems. Show your solution.

1. If the sum of two consecutive even numbers is 46, what are the numbers?
2. The perimeter of a rectangle is 30 cm. If its length is 8 cm, what is the width?
3. If a student's test scores are 80, 85, and 90, what is the average score?
4. Alice is 15 years old, and her brother is 5 years younger. How old is her brother?
5. A 20-liter container has a mixture of water and juice in a 3:2 ratio. How many liters of juice are in the mixture?
6. The shirt originally costs PhP 400, but it's on sale for 25% off. What is the sale price?
7. Sarah invests PhP 1000 in a savings account with a 5% annual interest rate. How much money will she have after one year?



Key Points

- In solving word problems, the following are the steps to follow:

  1. Identify your unknown and represent it using a variable.
  2. Translate English sentences to mathematical equations.
  3. Solve for the unknown variable using the properties we have learned.
  4. After getting a value for the unknown variable, check your answer by substituting it back to your first equation or inequality.
  5. Once you're sure of your answer, don't forget to always label it.

- Some of the types of problems wherein we can apply equations in one variable are:

  1. Number relations
  2. Perimeter
  3. Averages and scores
  4. Age
  5. Mixture
  6. Discounts, mark-ups, and pricing
  7. Investment



Do

Encircle the letter of the correct answer.

1. What is the area of a rectangle with length L and width W?

a. LW b. 2L + 2W c. L - W d. L ÷ W

2. In the expression 3(x + 2), what operation is being performed inside the parentheses?

a. Multiplication b. Addition c. Subtraction d. Division

3. Solve for z in the equation 4z + 7 = 31.

a. 6 b. 8 c. 9 d. 10

4. What is the solution to the equation 2x + 3 = 11?

a. x = 2 b. x = 4 c. x = 5 d. x = 6

5. Solve for z in the equation 5z + 8 = 33.

a. z = 5 b. z = 7 c. z = 9 d. z = 25

6. If 2(a - 4) = 10, what is the value of a?

a. a = 6 b. a = 7 c. a = 8 d. a = 9

7. A rectangle has a length of L and a width of 7 units. If the perimeter is 30 units, which equation can be used to find L?

a. 2L + 7 = 30 b. 2L + 14 = 30 c. 2L + 7 = 15 d. 2L + 14 = 15

8. The area of a square is 36 square units. What is the length of one side of the square?

a. 7 units b. 8 units c. 9 units d. 10 units

9. If a car travels at a speed of 60 miles per hour for 3 hours, how far does it travel?

a. 120 miles b. 160 miles c. 180 miles d. 200 miles

10. A recipe calls for 2 cups of sugar to make 24 cookies. How many cups of sugar are needed to make 36 cookies using the same recipe?

a. 2 cups b. 3 cups c. 4 cups d. 5 cups



Brainstorm

You will work in pairs. Your teacher will assign you a partner. Each pair will create three problems involving algebraic expressions and formulas on (1) mixture, (2) discounts, mark-ups, and pricing, and (3) investment. Exchange and find the value of an unknown in the problem, Record your findings and reasoning in your notebooks. After analyzing several expressions, discuss your results and reasoning with each other.



Design

Your teacher will divide the class into groups of 4 members each. Each member is assigned a word problem. Analyze the given word problem, show your solution, and present your work to your groupmates.

Problem 1: Three times a number plus 7 equals 25. What is the number?

Problem 2: A square has a perimeter of 20 inches. What is the length of one side?

Problem 3: In a basketball game, a player scored 12, 15, 20, and 18 points. What is the average score?

Problem 4: The sum of John and his sister's ages is 28. If John is 18, how old is his sister?

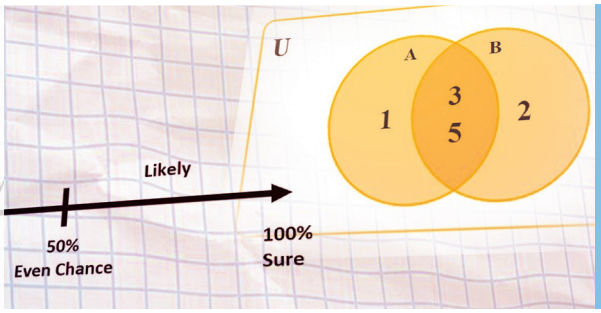


Fun Facts

The symbols commonly used in algebra, such as "x" for variables, were introduced by the French mathematician René Descartes in the 17th century. Before that, letters or words were used to represent unknown quantities in equations.

LESSON 5

Outcomes



Objectives

- Collect data from experiments.
- Express outcomes in words and symbols and represent outcomes in tables and graphs.

Examine

Here are some real-life situations involving outcomes. Predicting weather conditions involves collecting data on temperature, humidity, and wind speed. Graphs and tables displaying historical data are utilized to forecast future weather outcomes.

Tracking data on inflation rates, consumer spending, and employment rates assists economists in predicting economic outcomes and making financial decisions.

Researchers collect data on exercise habits, diet, and lifestyle choices to study their impact on health outcomes such as weight loss, blood pressure, or cholesterol levels. Are you eating healthy?

Coaches use data collected from sports and games to analyze player performance. Outcome data like scores, assists, or rebounds are used to identify patterns and improve team strategies. What can you improve on in your favorite sport?

Outcomes

Outcomes

Problem Solving Outcomes

Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).

Schools use exam results and performance data to evaluate teaching methods. Outcome data is utilized to identify areas for improvement and tailor instructional approaches. How would you improve your school performance?

In this lesson, we are to learn to gather data through experiments, such as coin tosses and die rolls, express the results using both descriptive language and symbols, and represent the outcomes systematically in tables and graphs, fostering skills in data collection and visualization.



Recall

What will you get if you toss a 1-peso coin?

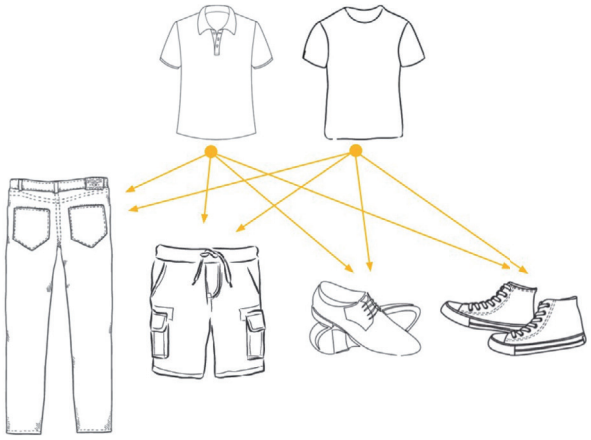
What are the board games you played that used a die?

What are the possibilities if you roll a die?



Explore

Juan was invited by his cousin to attend his birthday party. He only has a few choices left for shirts, pants, and shoes. How many different outfits can Juan choose?



An experiment is an activity in which the result cannot be predicted certainly. Choosing which t-shirt, pants, and pair of shoes for Juan is the experiment. There are two choices in each experiment. Tossing a coin is another perfect example of an experiment. Can we predict the result "heads" surely when we toss a coin? Of course not! Other examples are rolling a six-sided die and drawing a card from the standard deck of cards.

The result of the experiment is called an outcome. If tossing a coin is the experiment, then getting a result of "tails" is one of the possible outcomes. Getting a result of "heads" is the other possible outcome.

If we combine all the possible outcomes, we will form a sample space. It is the set of all possible outcomes generated from the experiment. From the preceding example in choosing an outfit, the "{T-Shirt, Pants, Shoes}" are all the possible results. From the example of tossing a coin, the {heads, tails} is the sample space.

There are some instances in which experiments have conditions. An example is rolling a die with a result of an even number. Rolling a die without any certain conditions generates a sample space of {1, 2, 3, 4, 5, 6}. If we insert the condition "with a result of even number", the possible outcomes will be {2, 4, 6}, and this is called the event. An event is a set of some possible outcomes that satisfy the given condition in an experiment. Also, an event is always a subset of a sample space.

There are some events or sample space of an experiment that cannot be determined by a simple listing. To deal with this situation, we may use the following methods: table, tree diagram, or systematic listing. There is no best method to use among these three. It always depends on the situation for which is more convenient to use.

The table is convenient to use in finding sample space when there are two objects used in an experiment. Also, it can be used when an experiment occurs twice. For example, list the sample space when a pair of dice is tossed. Since there are two dice used, we may use the table to identify the sample space.

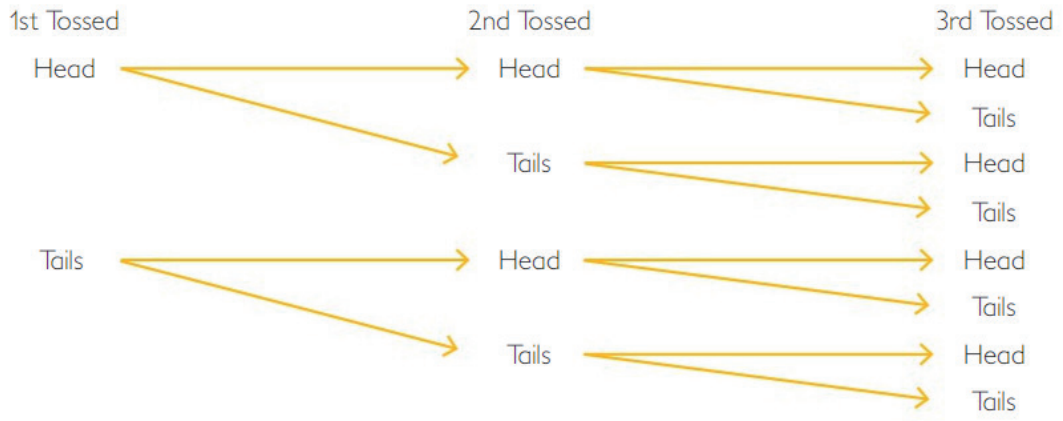
Using the table to show all the possible outcomes, we have:

Die 2 \ Die 1	1	2	3	4	5	6
1	1, 1	2, 1	3, 1	4, 1	5, 1	6, 1
2	1, 2	2, 2	3, 2	4, 2	5, 2	6, 2
3	1, 3	2, 3	3, 3	4, 3	5, 3	6, 3
4	1, 4	2, 4	3, 4	4, 4	5, 4	6, 4
5	1, 5	2, 5	3, 5	4, 5	5, 5	6, 5
6	1, 6	2, 6	3, 6	4, 6	5, 6	6, 6

Therefore, the sample space is {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}.

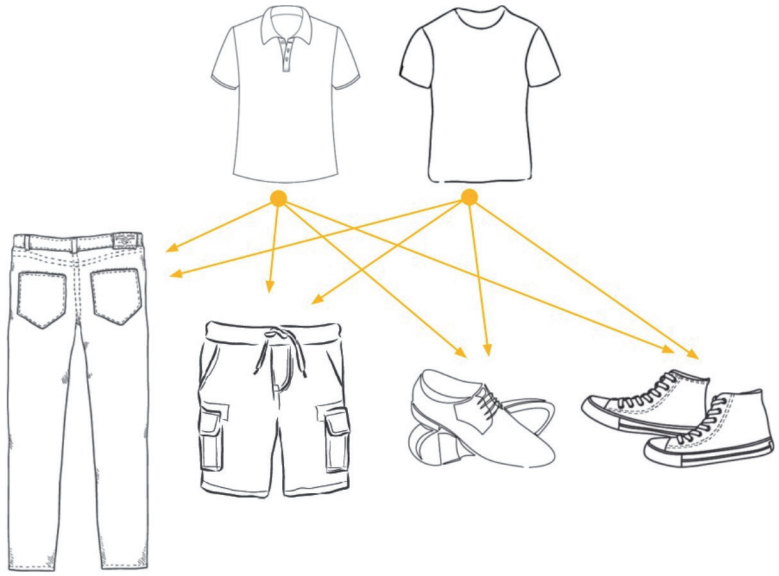
The tree diagram is practical to use in finding sample space when there are three or more objects involved or the experiment occurs more than twice. For example, list the sample space when a coin is tossed thrice. Since the tossing of the coin is done three times, we may use the tree diagram to identify the sample space.

Using the tree diagram to show all the possible outcomes, we have:



Therefore, the sample space is {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}.

Another example of this is the clothes that Juan will wear.



Therefore, the sample space is {(T-Shirt 1, Pants 1), (T-Shirt 1, Pants 2), (T-Shirt 1, Shoes 1), (T-Shirt 1, Shoes 2), (T-Shirt 2, Pants 1), (T-Shirt 2, Pants 2), (T-Shirt 2, Shoes 1), (T-Shirt 2, Shoes 2)}. There are 8 different outfits that Juan can choose from.

We may use the systematic listing if we want to find an event of an experiment that cannot be answered by simple listing. For example, the task is to list all three digits numbers less than 300 given the digits (0, 1, 2, 3, 4). Since we have to find the event, which is less than 300, and not just all three-digit numbers, we may use the systematic listing.

Less than 300 means the hundreds digit is either 1 or 2. The "0" cannot be put as the 1st digit of two or more-digit numbers. Using the systematic listing to determine the event, we have:

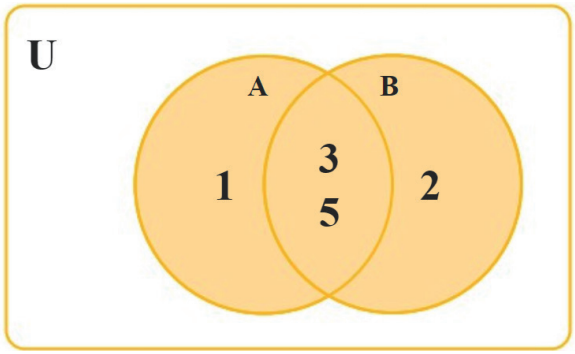
100, 110, 120, 130, 140, 200, 210, 220, 230, 240
101, 111, 121, 131, 141, 201, 211, 221, 231, 241
102, 112, 122, 132, 142, 202, 212, 222, 232, 242
103, 113, 123, 133, 143, 203, 213, 223, 233, 243
104, 114, 124, 134, 144, 204, 214, 224, 234, 244

Therefore, the event is {100, 101, 102, 103, 104, 110, 111, 112, 113, 114, 120, 121, 122, 123, 124, 130, 131, 132, 133, 134, 140, 141, 142, 143, 144, 200, 201, 202, 203, 204, 210, 211, 212, 213, 214, 220, 221, 222, 223, 224, 230, 231, 232, 233, 234, 240, 241, 242, 243, 244}.

The number of events in a sample space is not limited to one. There can be two or more events in a sample space. Also, an event is a subset of sample space. Speaking of set, we can talk about the union, intersection, and complement of events. We discussed this before in Quarter 2 and we can use it here.

The union of events refers to all outcomes in either or both events. The union of two events A and B is denoted by A ∪ B and read as "A union B". It may also be expressed as "A or B".

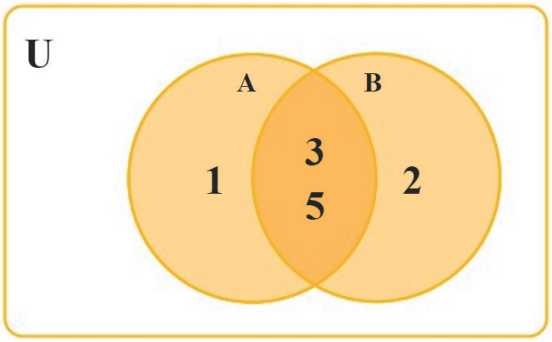
For example, we roll a fair six-sided die; let us represent A for the event of getting an odd number and B for the event of getting a prime number. Using the Venn diagram to visualize the union of events A and B for the experiment of rolling a fair six-sided die, we have:



Thus, A ∪ B consists of the set of outcomes of either A or B and both A and B. If A = {1, 3, 5} and B = {2, 3, 5}, then A ∪ B = {1, 2, 3, 5}.

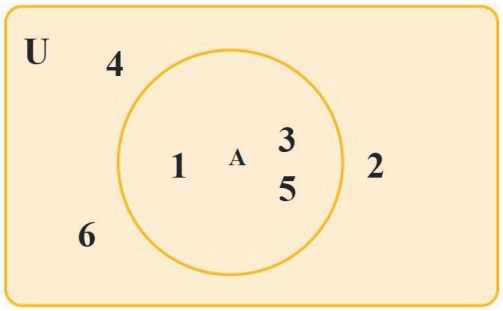
The intersection of two events is all the outcomes present in both events only. The intersection of two events A and B is denoted by A ∪ B read as "A intersection B". It may also be expressed as "A and B".

Using the Venn diagram to represent the intersection of events A and B for the experiment of rolling a fair six-sided die, we have:



Thus, if A = {1, 3, 5} and B = {2, 3, 5}, then A ∩ B = {3, 5}. The complement of an event A is the set of elements not in A but still an element found in sample space U. The complement of event A is denoted by A′ and read as "complement of A" or "A prime".

The Venn diagram shows representation of the complement of event A in the experiment of rolling a fair six-sided die.



Thus, the complement of A comprises all the outcomes in the sample space U that are not in A. If A = {1, 3, 5}, then A′ = {2, 4, 6}.



Try This

Work individually and answer the questions on the following situations.

1. If you toss a fair coin 20 times, how many times would you expect to get heads?
2. If you toss two fair coins simultaneously, how many possible outcomes are there?
3. How many prime numbers are there between 1 and 20?



Key Points

- An experiment is an activity in which the result cannot be predicted certainly.
- An outcome is the result of the experiment.
- The sample space is the set of all possible outcomes generated from the experiment.
- An event is a set of some possible outcomes that satisfies the given condition in an experiment.
- We may use a table, tree diagram, or systematic listing to determine the sample space and events.



Do

Encircle the letter of the correct answer.

1. How many faces does a standard cube (six-sided die) have?

a. 3 b. 4 c. 5 d. 6

2. In a series of 30 coin tosses, you recorded 15 heads. What percentage of the tosses resulted in heads?

a. 30% b. 45% c. 50% d. 60%

3. When conducting a coin toss experiment, you represented heads with "H" and tails with "T." How would you represent two heads in a row?

a. HT b. HH c. TT d. TH

4. In a coin-tossing experiment, what does each toss represent?

a. A random event
b. A predictable outcome
c. A mathematical equation
d. A controlled variable

5. In forming a two-digit number from the given set {0, 1, 2}, what is the sample space?

a. {10, 11, 12, 20, 21, 22}
b. {10, 12, 20, 21}
c. Forming a two-digit number
d. 4

6. Which of the following is true about A union B?
   a. It is the set of outcomes present in A
   b. It is the set of outcomes present in B
   c. It is the set of outcomes present in both A and B
   d. None of the above

7. The complement of an event means _____.
   a. The outcomes in sample space absent in that event
   b. The outcomes in sample space present in that event
   c. The outcomes in sample space
   d. The outcomes outside the sample space

8. It is the set of outcomes present in both X and Y.
   a. $X \cap Y$ b. $X \cup Y$ c. X' d. $X - Y$

9. Which of the following is an event for drawing a vowel from the letters h, a, p, p, i, e, s, t?
   a. {h, a, p, p, i, e, s, t}
   b. {h, p, p, s, t}
   c. {a, e, i}
   d. {h, a, p, p, y}

10. Which of the following is true about the sample space and event?
    a. The event is always a subset of the sample space
    b. The sample space is always the subset of the event
    c. The number of outcomes from a sample space and an event are always the same
    d. All the above



Brainstorm

Your teacher will find you a partner. You will work in pairs to list all the possible outcomes for the following experiments. You will record their findings and reasoning in their notebooks. After analyzing several expressions, you will discuss with your partner.

1. Picking 1 ball out of 3 balls with primary colors
2. Picking 1 ball out of 3 balls with secondary colors
3. Choosing 1 planet out of 8 planets of the Solar System
4. Drawing a flower from the standard deck of cards
5. Drawing a king from the standard deck of cards



Design

Your teacher will divide the class into groups with 4 members each. Each person is assigned to determine the sample space or events in the form of a table, tree diagram, or systematic listing. Discuss your answers after.

Person 1: Choosing an Android phone that is Samsung, Oppo, Vivo, Huawei, Cherry Mobile, or My Phone brand that is 3G or LTE

Person 2: Choosing a viand: Sinigang, Bulalo, or Nilaga; selecting a drink: Buko, Orange, or Mango Juice; and getting a dessert: Leche Flan or Fruit Salad

Person 3: Tossing a coin and a fair six-sided die

Person 4: From the given sets {0, 1, 2, 3, 4}, form three-digit numbers less than 200

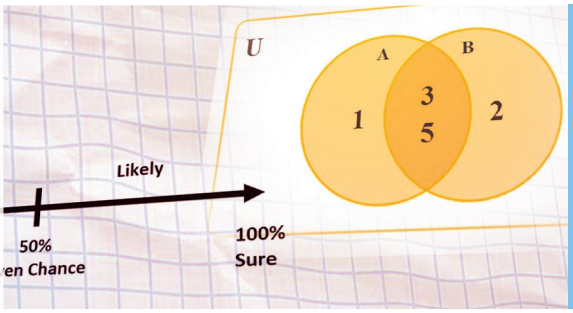


Fun Facts

Outcomes and probability play a role in popular board games like dice-rolling games, where understanding outcomes helps in strategizing moves.

LESSON 6

Problem Solving Outcomes



Objectives

- Solve problems using the outcomes of experiments.

Examine

Here are some more real-life situations involving outcomes. Meteorologists predict weather outcomes using data analysis, aiding in planning activities and preparing for different weather conditions. Researchers use outcomes to understand the impact of human activities on the environment, aiding in the development of conservation strategies. Understanding potential outcomes in video games helps players develop strategies and make informed decisions during gameplay. What field are you interested in?

In this lesson, we are to focus on applying the results obtained from experiments to solve various types of problems, enhancing our critical thinking and problem-solving skills through practical application.

Outcomes

Outcomes

Problem Solving Outcomes

Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).



Recall

Here are some important questions to ponder before our discussion.

How can understanding outcomes from past experiments or events help in making informed decisions for the future?

In what ways can representing outcomes using tables, graphs, or diagrams assist in understanding and interpreting data more effectively?



Explore

The teacher told the students in the class his basic requirements to get a passing grade in Mathematics 8. Some students are already confident that they will pass the subject. Others are still hoping for a chance to pass the subject.

The teacher's basic requirements for the students to pass the Math subject are the following:

- Get at least half of the total score in quizzes.
- Get at least 95% of the total attendance.
- Recite in class.
- Submit the project before the date of deadline.
- Do the activities.
- Show respect to the teacher and students.
- Get at least half of the total score in periodical exam.

If a student only attends the classes but does not accomplish the others, what do you think is the chance that he/she passes the subject?

The situation implies the chance of the student to pass the math subject. The question "If a student only goes to school but does not accomplish the others, what do you think is the chance that he/she passes the subject?" could be answered mathematically by means of probability. Probability is the branch of mathematics that deals with the study of chances of occurrence. There are many types of probability. One type is theoretical probability.

Theoretical probability is one of the types of probability that deals with what is expected to happen. It came from two words which are "theoretical" and "probability". The definition of probability was already stated which is the study of chances of occurrence. The word "theoretical" pertains to the principle that is highly possible true.

The formula for computing the theoretical probability of an experiment is given.

$$\text{Theoretical Probability} = \frac{\text{Number of favorable outcomes (event)}}{\text{Number of all possible outcomes (sample space)}}$$

In the situation, it is stated that there are 7 requirements that should be accomplished by the students. If a student accomplished 1 requirement which is attending the class only, then his/her chance is 1 out 7. The extended discussion is on the probability of simple experiment. Before that, here is a brief study about simple experiments.

A simple experiment has only one object chosen or has only one trial in the experiment. If the number of outcomes in the event or sample space can be easily determined by listing and counting one by one, then the activity done is a simple experiment. One of the examples of a simple experiment is tossing a coin. There is only one object which is the coin. The experiment is done only in one trial which is tossing a coin once. If the example is tossing two coins, then it is not a simple experiment. Although the experiment is done in one trial, there are two objects involved which are the two coins. Tossing a coin twice is also not an example of a simple experiment. There are two trials of the experiment done even if there is only one object.

The probability of a simple experiment pertains to the chance of getting the outcome from the event out of all the possible outcomes in the sample space from the simple experiment. One of the examples, where you can apply the probability of a simple experiment, is finding the chance of getting heads when you tossed a coin. Tossing a coin is an example of a simple experiment as you learned earlier. To find the chance of getting heads we will use the formula of theoretical probability. There are 2 total possible outcomes in tossing a coin which are heads and tails. Meanwhile, there is only 1 heads in tossing a coin. Writing the solution, we have:

$$\text{Chance of getting heads} = \frac{1}{2}$$

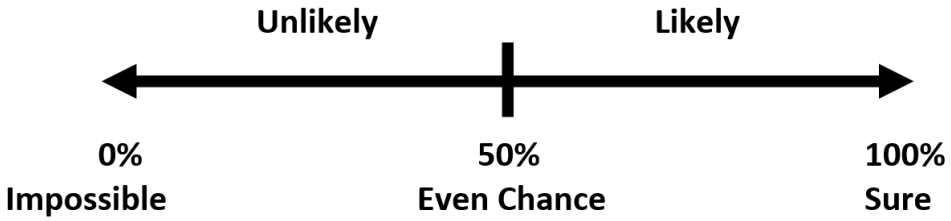
The answer should always be reduced in simplest form. Therefore, the probability of getting heads is equal to $\frac{1}{2}$.

Let us go back to the situation of the math class. There are 7 requirements that should be accomplished by the students. A student accomplishes 1 requirement which is attending the class only. Writing the solution, we have:

$$\text{Chance of passing} = \frac{1}{7}$$

Therefore, the chance of the student passing the math class is equal to $\frac{1}{7}$.

The probability line is used to interpret the probability result. The interpretation of the probability result is necessary to find out whether it is impossible, unlikely, even chance, likely, or sure to happen. The figure shows the probability line.



Before the interpretation, the numerical result of probability should be converted first into percent. The description of the interpretation is shown:

Impossible to happen	-	The probability result is 0%.
Unlikely to happen	-	The probability result is between 0% and 50%.
Even chance to happen	-	The probability result is 50%.
Likely to happen	-	The probability result is between 50% and 100%.
Sure to happen	-	The probability result is 100%.

If we go back to the topic of the probability of the simple experiment, the half-chance of getting heads is equivalent to 50%. It means that getting heads in tossing a coin has an even chance to happen.

Another example is the $\frac{1}{7}$ chance of a student to pass the math subject if he/she only attends the class but does not accomplish the other requirements. This numerical value is approximately equal to 14.29%. It means that the chance to pass is unlikely to happen. Studying is still the key to getting a passing grade.

Experimental probability is one of the types of probability that deals with what actually happens when we try out the activity. One of the famous examples is the probability of getting heads and the chance of getting tails in tossing a fair coin. In terms of theoretical probability, the probability of getting heads is $\frac{1}{2}$ and the chance of getting tails is also the same. If we put this in terms of experimental probability, we will toss the coin many times, and then find the frequency of the heads and tails. For example, we toss the coin 50 times, and we got 20 heads and 30 tails. The probability of getting heads will be 20 out of 50 or $\frac{2}{5}$. The chance of getting tails will be 30 out of 50 or $\frac{3}{5}$.

Experimental probability is more useful when you want to know the chances of occurrence in a more specific situation. It is more realistic than theoretical probability.

Let's have this example.

The school canteen committee conducted a survey tally of 100 sample students about their favorite snacks to eat. Their objective is to find out the chance that students will buy the food on the list. The student will vote for only one food from the list. The food choices are burgers, turon, cookies, and spaghetti.

After the survey, they made a frequency table that shows the number of occurrences of each food.

Based on the frequency table, find the chance that the students will buy burgers, turon, cookies, and spaghetti.

Food	Frequency
Burgers	17
Turon	30
Cookies	20
Spaghetti	33
Total	100

The situation implies the chance that the student will buy a certain food. In the theoretical probability aspect, the probability that the student will buy the burger sandwich for example is $\frac{1}{4}$. There are a total of 4 kinds of foods available in the canteen and the student will choose only one food. If the aspect is based on the survey, the probability value will be different. The number of outcomes in events of each food will be based on the frequency of that particular food from the experiment. The number of outcomes in the sample space will be based on the total frequency. There are 17 students that choose the burgers and there are a total of 100 students. Therefore, the chance that the student will buy the following is:

$$\text{Burgers} = \frac{17}{100}$$

$$\text{Cookies} = \frac{20}{100} = \frac{1}{5}$$

$$\text{Turon} = \frac{30}{100} = \frac{3}{10}$$

$$\text{Spaghetti} = \frac{33}{100}$$



Try This

Find the probability of the following situations.

1. You roll a standard six-sided die. What is the probability of rolling a 4?
2. In an experiment, you spin a spinner with four equal sections labeled A, B, C, and D. What is the probability of landing on section A?
3. In a bag, there are 5 green marbles, 3 blue marbles, and 2 red marbles. What is the probability of randomly selecting a blue marble?



Key Points

- Theoretical probability is one of the types of probability that deals with what is expected to happen.
- A simple experiment has only one object chosen or has only one trial in the experiment.
- The probability of a simple experiment pertains to the chance of getting the outcomes from the event out of all the possible outcomes in the sample space from the simple experiment.
- The probability line is used to interpret the probability result. The interpretation of probability result is necessary to find out whether it is impossible, unlikely, even chance, likely, or sure to happen.
- Experimental probability is one of the types of probability that deals with what happens when we try out the activity.



Do

Encircle the letter of the correct answer.

1. It is a type of probability that deals with what happens when we try out the activity.

a. Theoretical Probability
b. Experimental Probability
c. Conditional Probability
d. Personal Probability

2. On a probability line, what is typically represented by the midpoint?

a. Impossible event
b. Certain event
c. Unlikely event
d. Equally likely event

3. If an event is very likely to occur, where on the probability line would you place it?

a. Near the left end
b. Near the right end
c. In the middle
d. It depends on the context

4. In a standard deck of 52 playing cards, how many possible outcomes are there when drawing a single card?

a. 13 b. 26 c. 52 d. 4

5. If you flip a fair coin, how many equally likely outcomes are there?

a. 1 b. 2 c. 3 d. 4

6. If an experiment has two equally likely outcomes (success or failure), what is the theoretical probability of success?

a. 0 b. $\frac{1}{2}$ c. 1 d. 2

7. In an experiment, a fair six-sided die is rolled, and the results are recorded. After 100 rolls, the number 3 came up 20 times. What is the experimental probability of rolling a 3?

a. $\frac{1}{6}$ b. $\frac{1}{5}$ c. $\frac{1}{4}$ d. $\frac{1}{2}$

8. A spinner with four equal sections is spun 80 times in an experiment. The results show that it landed on section A 20 times, section B 30 times, section C 15 times, and section D 15 times. What is the experimental probability of landing on section B?

a. $\frac{1}{4}$ b. $\frac{3}{8}$ c. $\frac{3}{40}$ d. $\frac{2}{5}$

9. In a bag, there are 5 red marbles, 3 blue marbles, and 2 green marbles. If you randomly select a marble 50 times with replacement, and you get a red marble 30 times, what is the experimental probability of selecting a red marble?

a. $\frac{3}{10}$ b. $\frac{2}{5}$ c. $\frac{1}{2}$ d. $\frac{3}{5}$

10. In an experiment, you toss a coin 120 times. It lands on heads 70 times and tails 50 times. What is the experimental probability of getting heads?

a. $\frac{7}{12}$ b. $\frac{3}{4}$ c. $\frac{2}{5}$ d. $\frac{5}{12}$



Brainstorm

Your teacher will find you a partner and you will work in pairs to indicate whether the following probabilities are impossible, unlikely, even chance, likely, or sure to happen.

1. 26% **2.** 95% **3.** 0.0000 **4.** 1.0000 **5.** $\frac{10}{20}$



Design

Your teacher will divide your class into groups with 3 members each. Each member will be given a task to answer. Assign yourselves a number from 1 to 3. Shown is a frequency table of the number of occurrences of each drawn card from a standard deck. Before drawing another card, return first the card from the previous draw. Compare your answers once every one is done.

Card	Frequency
Diamond	13
Heart	6
Spade	19
Club	12
Total	50

Person 1:

1. Based on the data in the frequency table, what is the probability of getting a diamond?
2. Suppose that we put the given problem in the context of theoretical probability, which is the drawing of a card from a standard deck. What is the probability of getting a diamond?
3. Compare the values of the experimental and theoretical probabilities. What can you say about the difference?

Person 2:

1. Based on the data in the frequency table, what is the probability of getting a heart?
2. Suppose that we put the given problem in the context of theoretical probability, which is the drawing of a card from a standard deck. What is the probability of getting a heart?
3. Compare the values of the experimental and theoretical probabilities. What can you say about the difference?

Person 3:

1. Based on the data in the frequency table, what is the probability of getting a spade?
2. Suppose that we put the given problem in the context of theoretical probability, which is the drawing of a card from a standard deck. What is the probability of getting a spade?
3. Compare the values of the experimental and theoretical probabilities. What can you say about the difference?

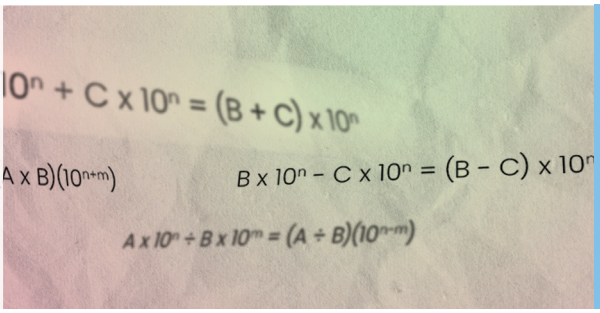


Fun Facts

The concept of sample space and events has been already used as far as 2800 BC. The Chinese use this to solve and create the magic square. It is an array of number in a square such that the sum of the rows, columns, and slants are all equal.

LESSON 7

Scientific Notation





Objectives

- Write numbers in scientific notation to represent very large or very small numbers, and vice versa.
- Perform operations on numbers expressed in scientific notation.



Examine

Here are some real-life situations involving scientific notation. Understanding and expressing the population count of cities or provinces in the Philippines, where large numbers are often represented more concisely using scientific notation. What is the population in your city?

Discussing the country's Gross Domestic Product (GDP), where scientific notation is used to represent large monetary values in billions or trillions. What is the current GDP of the Philippines?

Scientific Notation

Scientific Notation



Interactives

To learn more about this lesson, you may visit TekTeach (Learning Management System).

Exploring the vast distances between islands or provinces, where scientific notation can simplify expressions of distances in kilometers. What is the distance between where you are now and the place you were born?

Representing the number of mobile phone users or subscribers of a network in the Philippines in millions or billions using scientific notation.

Expressing seismic magnitudes or volcanic eruption intensity, where scientific notation is used to denote the magnitude of an earthquake.

In this lesson, we are to learn the essential skills of converting numbers into scientific notation for convenient representation of extremely large or small values and performing mathematical operations seamlessly on numbers expressed in scientific notation, fostering numerical fluency and problem-solving capabilities in scientific contexts.



Recall

Encircle the letter of the correct answer.

1. Express 5 600 000 in scientific notation.

a. 5.6×10^6 b. 56×10^5 c. 560×10^4 d. 0.56×10^7

2. Write 0.000 047 in scientific notation.

a. 4.7×10^{-5} b. 47×10^{-6} c. 470×10^{-7} d. 0.47×10^{-5}

3. Subtract 2.7×10^6 from 4.9×10^6.

a. 2.2×10^5 b. 2.2×10^6 c. 2.2×10^7 d. 22×10^5

4. When multiplying two numbers in scientific notation, what should you do with the exponents?

a. Add them b. Subtract them c. Multiply them d. Divide them

5. Which of the following numbers is the smallest in scientific notation?

a. 3×10^5 b. 3×10^4 c. 3×10^3 d. 3×10^2

6. What is the quotient of 9.6×10^7 divided by 3.0×10^3?

a. 3.2×10^4 b. 3.2×10^5 c. 3.2×10^6 d. 3.2×10^7

7. Which of the following numbers is correctly written in scientific notation?

a. 5 000 000 b. 50×10^4 c. 5×10^6 d. 0.5×10^{-6}

8. You are given two numbers in scientific notation: (2.3×10^4) and (5.0×10^{-2}). What is the product of these two numbers?

a. 11.5×10^2 b. 1.15×10^3 c. 11.5×10^{-2} d. 1.15×10^{-1}

9. You have two numbers in scientific notation: 6.4×10^5 and 2.5×10^{-3}. What is the result of dividing the first number by the second number?

a. 2.56×10^8 b. 256×10^4 c. 2560×10^2 d. 2.56×10^5

10. The distance from Earth to the Moon is approximately 3.8×10^5 kilometers. If a spacecraft travels at a speed of 2.0×10^4 kilometers per hour, how many hours will it take to reach the Moon?

a. 1.9×10^1 hours b. 1.9×10^2 hours c. 1.9×10^3 hours d. 1.9×10^4 hours



Explore

Scientific notation is defined to represent any number as a product of a real number and a power of 10. Scientific notation is the way that scientists easily handle very large numbers or very small numbers. In fact, you'll quickly see that working in scientific notation enables us to work effectively all while avoiding careless mistakes with decimals. To begin, we must understand how to read and write a number in scientific notation. When a number is written in scientific notation, the exponent tells you if the term is a large or a small number. A positive exponent indicates a large number and a negative exponent indicates a small number that is between 0 and 1.

Here is how to write a number in scientific notation.

Step 1: Move the decimal point to the right or left until after the first significant digit and copy the significant digits to the right of the first digit. If the number is a whole number and has no decimal point, place a decimal point after the first significant digit and copy the significant digits to its right.

400 000 000 has 1 significant digit, which is 4.

Place a decimal point after 4 so, 4.0

0.000 000 097, its first significant digit is 9.

We place a decimal point after 9, so 9.7.

Step 2: Multiply the adjusted number in step 1 by a power of 10, the exponent of which is the number of digits that the decimal point was moved, positive if moved to the left and negative if moved to the right. 400 000 000 is written as 4.0×10^8.

The decimal point was moved 8 places to the left which is why the exponent 8 is positive.

0.000 000 089 is written as 8.9×10^{-8}.

The decimal point was moved 8 places to the right which is why exponent 8 is negative.

Here are the operations on scientific notation.

To add or subtract two or more numbers in scientific notation, these numbers must have the same exponent for 10. That is,

$$B \times 10^n + C \times 10^n = (B + C) \times 10^n$$

$$B \times 10^n - C \times 10^n = (B - C) \times 10^n$$

Here is an example.

$$5.89 \times 10^3 + 9.234 \times 10^5$$

We need to have the same exponent for base 10. Let's reduce the power from 5 to power 3.

$9.234 \times 10^5 = 923.4 \times 10^3$ so we can add it to 5.89×10^3

$$5.89 \times 10^3 + 923.4 \times 10^3 = (5.89 + 923.4) \times 10^3 = 929.29 \times 10^3$$

Here is another example.

$$5.34 \times 10^4 - 25.390\ 42 \times 10^2 = 5.34 \times 10^4 - 0.253\ 904\ 2 \times 10^4 = (5.34 - 0.253\ 904\ 2) \times 10^4$$

$$= 5.086\ 095\ 8 \times 10^4$$

To multiply two numbers in scientific notation, get the product of the numerical coefficients x 10 raised to the sum of the exponents. That is,

$$(A \times 10^n)(B \times 10^m) = (A \times B)(10^{n+m})$$

Here is an example.

$$(3.5 \times 10^3)(1.35 \times 10^2) = (3.5 \times 1.35) \times 10^{2+3} = 4.725 \times 10^5$$

To divide two numbers in scientific notation, get the quotient of the numerical coefficients x 10 raised to the difference of exponents. That is,

$$A \times 10^n \div B \times 10^m = (A \div B)(10^{n-m})$$

Here is an example.

$$300 \times 10^8 \div 50 \times 10^{-3} = (300 \div 50) \times 10^{8-(-3)} = 6 \times 10^{8+3} = 6 \times 10^{11}$$



Try This

Work individually to answer the following.

I. Express the following in scientific notation.

1. 6 250 000

2. 0.000 003 48

II. Perform the given mathematical operation.

1. $(8.6 \times 10^{2})(1.5 \times 10^{1})$

2. $9.5 \times 10^{5} - 2.7 \times 10^{5}$



Key Points

- Scientific notation is defined to represent any number as a product of real number and a power of 10.
- To add or subtract two or more numbers in scientific notation, these numbers must have the same exponent for 10. $B \times 10^{n} + C \times 10^{n} = (B + C) \times 10^{n}$ and $B \times 10^{n} - C \times 10^{n} = (B - C) \times 10^{n}$.
- To multiply two numbers in scientific notation, get the product of the numerical coefficients x 10 raised to the sum of the exponents. $(A \times 10^{n})(B \times 10^{m}) = (A \times B)(10^{n+m})$
- To divide two numbers in scientific notation, get the quotient of the numerical coefficients x 10 raised to the difference of exponents. $A \times 10^{n} \div B \times 10^{m} = (A \div B)(10^{n-m})$



Do

I. Encircle the letter of the correct answer.

1. Write the number 0.000 045 7 in scientific notation.

a. 4.57×10^{-5} b. 45.7×10^{-4} c. 457×10^{-6} d. 0.457×10^{4}

2. Express 8200 in scientific notation.

a. 82×10^{3} b. 8.2×10^{3} c. 8.2×10^{4} d. 0.82×10^{4}

3. What is the result of $(2.5 \times 10^{3})(3.2 \times 10^{2})$?

a. 8.0×10^{5} b. 8.0×10^{6} c. 80×10^{5} d. 80×10^{6}

4. Which of the following numbers is written in scientific notation?

a. 3500 b. 35×10^2 c. 0.0035 d. 3.5×10^3

5. When dividing two numbers in scientific notation, what should you do with the exponents?

a. Add them b. Subtract them c. Multiply them d. Divide them

6. What is the scientific notation for the number 0.000 025?

a. 2.5×10^5 b. 0.0025×10^{-2} c. 25×10^{-6} d. 2.5×10^{-5}

7. What is the exponent in the scientific notation representation of 6.72×10^4?

a. 6 b. 4 c. 0.72 d. 10

8. Write 0.000 065 in scientific notation.

a. 65×10^{-4} b. 6.5×10^{-5} c. 65×10^{-5} d. 6.5×10^{-4}

9. What is the product of 3.0×10^3 and 2.5×10^2?

a. 7.5×10^5 b. 7.5×10^6 c. 75×10^5 d. 75×10^6

10. Calculate the result of $2.5 \times 10^5 + 4.0 \times 10^4$.

a. 2.9×10^5 b. 2.1×10^5 c. 6.5×10^4 d. 2.9×10^4

II. Tell whether the following is written in scientific notation or not. Write YES if they are written in scientific notation otherwise write NO.

1. 4.5×10^{10}

2. 12

3. 0.1

4. 0.01

5. 5.12×10^{-15}

6. 4.10×10^{-3}

7. 70 000

8. 5.0

9. 1 000 000 000

10. 5.3×10^7

III. Express the following numbers into standard scientific notation.

1. 345.25

2. 0.000 456

3. 187 645 890

4. 0.023 700 035 6

IV. Express the following numbers in scientific notation to ordinary numbers:

1. 2.34589×10^{-8}

2. 1.004567×10^{-5}

3. 2.3456×10^4

V. Perform addition or subtraction in each of the following.

1. $5.933\ 35 \times 10^{6} + 9.598 \times 10^{5} =$
2. $78 \times 10^{8} - 0.45 \times 10^{10} =$
3. $5.4 \times 10^{12} + 584.894 \times 10^{8} =$
4. $2.09 \times 10^{-5} - 8.45 \times 10^{-6} + 4.8930 \times 10^{-2} =$

VI. Perform multiplication or division in each of the following.

1. $1.676 \times 10^{-3} \times 29.45 \times 10^{-5} =$
2. $3.5 \times 10^{4} \times 150 \times 10^{3} =$
3. $6 \times 10^{24} \div 12.25 \times 10^{20} =$
4. $3.580 \times 10^{-3} \div 8.95 \times 10^{-4} =$



Brainstorm

Your teacher will find you a partner. Share your answers with your pair to further explain your thought processes on how to perform operations on scientific notations.

1. Add the numbers in scientific notation: $5.2 \times 10^{6} + 3.8 \times 10^{6}$
2. Subtract the numbers in scientific notation: $7.2 \times 10^{4} - 2.5 \times 10^{4}$
3. Multiply the numbers in scientific notation: $(3.2 \times 10^{4})(2.5 \times 10^{3})$
4. Divide the numbers in scientific notation: $6.4 \times 10^{5} \div 4 \times 10^{2}$



Design

Your teacher will divide the class into groups with 4 members each. You will each create 1 very large number and write it scientific notation and 1 very small number and write it in scientific notation. Once you are all done, discuss with one another how you converted to scientific notation.



Fun Facts

A light year is the number of miles light travels in one year, about 5 880 000 000 000. That's a lot of zeros, and it is easy to lose count when trying to figure out the place value of the number. Using scientific notation, the distance is 5.88×10^{12} miles.

Unit Performance Task: Algebra Workbook and Data Project

Your teacher will group you into teams of 4 members each. Work together to do the following.

Component 1: Algebraic Expressions and Equations

Create an algebraic expressions and equations workbook.

Include 4 problems and solutions using bar models to find unknowns.

Provide 4 exercises to distinguish variables from constants and 4 to evaluate algebraic expressions.

Translate 4 verbal phrases to algebraic expressions and solve them.

Illustrate the properties of equality through 4 examples.

Provide 4 problems each where students solve for one variable in terms of another and find unknowns in equations.

Component 2: Experimental Data Collection and Analysis

Conduct an experimental data analysis project.

Design and perform 4 simple experiments (e.g., coin tosses, dice rolls) to collect data.

Express outcomes in words/symbols, and represent them in tables/graphs.

Solve 4 problems based on experimental outcomes.

Include 4 exercises on writing numbers in scientific notation and performing operations with them.

Class Presentation

Groups will present their algebraic expressions and equations workbook, explaining their methods and solutions.

Demonstrate their experimental data analysis project, showcasing their data collection, representation, and problem-solving techniques.

Each presentation should include visual aids (workbook pages, graphs) and be interactive.

Rubric

Criteria	Excellent (4)	Proficient (3)	Developing (2)	Beginning (1)
Algebraic Skills	Excellent problem-solving with bar models; clear distinction between variables/constants; accurate evaluation and translation of expressions.	Good problem-solving skills; mostly correct distinction and evaluation; adequate translation.	Basic problem-solving; some errors in distinction, evaluation, and translation.	Poor problem-solving; incorrect distinction, evaluation, and translation.
Application of Equality & Equations	Correct and insightful use of equality properties; accurate solutions for unknowns and variable relationships.	Mostly correct use of equality; good solutions with minor errors.	Basic understanding of equality; some correct solutions with errors.	Poor understanding of equality; incorrect or no solutions.
Data Collection & Analysis	Effective data collection; excellent expression and representation of outcomes; accurate problem-solving.	Good data collection; clear expression and representation; mostly accurate problem-solving.	Basic data collection; some errors in expression, representation, and problem-solving.	Ineffective data collection; poor expression, representation, and problem-solving.
Scientific Notation	Accurate use and operations with scientific notation; clear understanding and application.	Mostly accurate use; good understanding with minor errors.	Basic use and understanding; some errors in operations.	Incorrect use; poor understanding and application.
Presentation & Clarity	Engaging, clear presentation with thorough explanations and confident responses.	Good presentation; clear explanations and good responses.	Adequate presentation; some unclear explanations and hesitant responses.	Poor presentation; unclear explanations and weak responses.